

## DOCUMENT RESUME

ED 424 116

SE 061 864

AUTHOR Hawkins, Evelyn F.; Stancavage, Frances B.; Dossey, John A.  
TITLE School Policies and Practices Affecting Instruction in Mathematics: Findings from the National Assessment of Educational Progress.  
INSTITUTION Educational Testing Service, Princeton, NJ.; American Institutes for Research, Washington, DC.  
SPONS AGENCY National Center for Education Statistics (ED), Washington, DC.  
REPORT NO NCES-98-495  
ISBN ISBN-0-16-049716-7  
PUB DATE 1998-08-00  
NOTE 135p.  
AVAILABLE FROM ED Pubs, P.O. Box 1398, Jessup, MD 20794-1398; Tel: 877-433-7827 (Toll Free); Web site: <http://nces.ed.gov/naep/>; U.S. Government Printing Office, Superintendent of Documents, Mail Stop, SSOP, Washington, DC 20402-9328.  
PUB TYPE Reports - Research (143)  
EDRS PRICE MF01/PC06 Plus Postage.  
DESCRIPTORS \*Academic Achievement; \*Educational Assessment; \*Educational Policy; Elementary Secondary Education; \*Mathematics Instruction; National Competency Tests  
IDENTIFIERS National Assessment of Educational Progress

## ABSTRACT

The teaching and learning of mathematics continues to generate tremendous attention, both among those who support recent innovations and, more recently, among those who question the wisdom of the promulgated reforms. In order to bring an empirical basis to this debate, it is important to gather information on the policies and practices that are actually implemented. This report provides one source for such information and is the second in a series that discusses results from the National Assessment of Educational Progress (NAEP). A description of the educational policies and practices that prevailed during this period of sustained increases in mathematics achievement, giving particular attention to the relationship between these policies, practices, and student performance on the NAEP mathematics assessment. Information on the status of mathematics education in 1996 is provided, and changes that took place from the time of earlier NAEP assessments is also chronicled. (ASK)

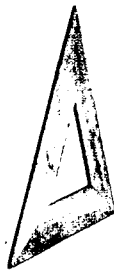
\*\*\*\*\*  
\* Reproductions supplied by EDRS are the best that can be made \*  
\* from the original document. \*  
\*\*\*\*\*

SE

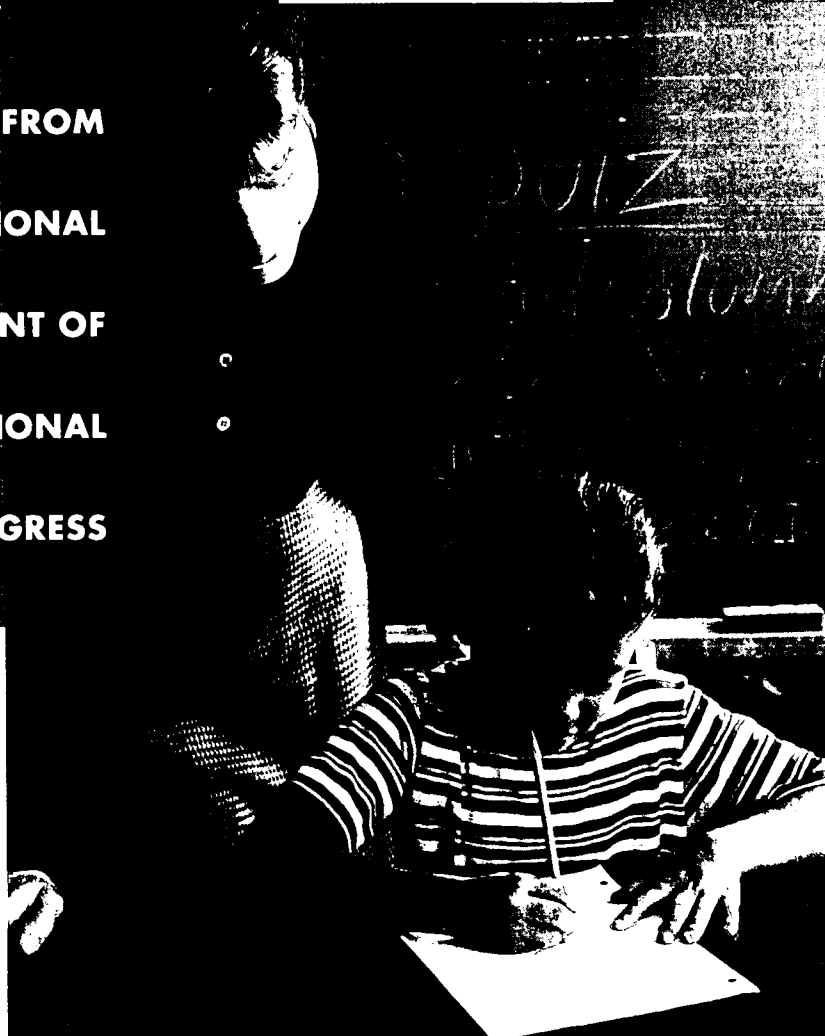
ED 424 116

NATIONAL CENTER FOR EDUCATION STATISTICS

# SCHOOL POLICIES AND PRACTICES AFFECTING INSTRUCTION IN MATHEMATICS



**FINDINGS FROM  
THE NATIONAL  
ASSESSMENT OF  
EDUCATIONAL  
PROGRESS**



U.S. DEPARTMENT OF EDUCATION  
Office of Educational Research and Improvement  
EDUCATIONAL RESOURCES INFORMATION  
CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

BEST COPY AVAILABLE

THE NATION'S  
REPORT  
CARD



U.S. Department of Education  
Office of Educational Research and Improvement

Full Text Provided by ERIC

## **What is The Nation's Report Card?**

THE NATION'S REPORT CARD, the National Assessment of Educational Progress (NAEP), is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas. Since 1969, assessments have been conducted periodically in reading, mathematics, science, writing, history/geography, and other fields. By making objective information on student performance available to policymakers at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement is collected under this program. NAEP guarantees the privacy of individual students and their families.

NAEP is a congressionally mandated project of the National Center for Education Statistics, the U.S. Department of Education. The Commissioner of Education Statistics is responsible, by law, for carrying out the NAEP project through competitive awards to qualified organizations. NAEP reports directly to the Commissioner, who is also responsible for providing continuing reviews, including validation studies and solicitation of public comment, on NAEP's conduct and usefulness.

In 1988, Congress established the National Assessment Governing Board (NAGB) to formulate policy guidelines for NAEP. The Board is responsible for selecting the subject areas to be assessed from among those included in the National Education Goals; for setting appropriate student performance levels; for developing assessment objectives and test specifications through a national consensus approach; for designing the assessment methodology; for developing guidelines for reporting and disseminating NAEP results; for developing standards and procedures for interstate, regional, and national comparisons; for determining the appropriateness of test items and ensuring they are free from bias; and for taking actions to improve the form and use of the National Assessment.

## **The National Assessment Governing Board**

**Mark D. Musick, Chair**  
President  
Southern Regional Education Board  
Atlanta, Georgia

**Mary R. Blanton, Vice Chair**  
Attorney  
Salisbury, North Carolina

**Patsy Cavazos**  
Principal  
W.G. Love Accelerated School  
Houston, Texas

**Catherine A. Davidson**  
Secondary Education Director  
Central Kitsap School District  
Silverdale, Washington

**Edward Donley**  
Former Chairman  
Air Products & Chemicals, Inc.  
Allentown, Pennsylvania

**James E. Ellingson**  
Fourth-Grade Classroom Teacher  
Probstfield Elementary School  
Moorhead, Minnesota

**Honorable John M. Engler**  
Member Designate  
Governor of Michigan  
Lansing, Michigan

**Thomas H. Fisher**  
Director, Student Assessment Services  
Florida Department of Education  
Tallahassee, Florida

**Michael J. Guerra**  
Executive Director  
National Catholic Education Association  
Secondary School Department  
Washington, DC

**Edward H. Haertel**  
Professor, School of Education  
Stanford University  
Stanford, California

**Lynn Marmer**  
President  
Cincinnati Board of Education  
Cincinnati, Ohio

**William J. Moloney**  
Commissioner of Education  
State of Colorado  
Denver, Colorado

**Honorable Annette Morgan**  
Former Member  
Missouri House of Representatives  
Jefferson City, Missouri

**Mitsugi Nakashima**  
First Vice-Chairperson  
Hawaii State Board of Education  
Honolulu, Hawaii

**Michael T. Nettles**  
Professor of Education & Public Policy  
University of Michigan  
Ann Arbor, Michigan  
and Director  
Frederick D. Patterson Research Institute  
United Negro College Fund

**Honorable Norma Paulus**  
Superintendent of Public Instruction  
Oregon State Department of Education  
Salem, Oregon

**Honorable Jo Ann Pottorff**  
Kansas House of Representatives  
Wichita, Kansas

**Honorable William T. Randall**  
Former Commissioner of Education  
State of Colorado  
Denver, Colorado

**Diane Ravitch**  
Senior Research Scholar  
New York University  
New York, New York

**Honorable Roy Romer**  
Governor of Colorado  
Denver, Colorado

**Fannie L. Simmons**  
Mathematics Coordinator  
District 5 of Lexington/Richland County  
Ballentine, South Carolina

**Adam Urbanski**  
President  
Rochester Teachers Association  
Rochester, New York

**Deborah Voltz**  
Assistant Professor  
Department of Special Education  
University of Louisville  
Louisville, Kentucky

**Marilyn A. Whirry**  
Twelfth-Grade English Teacher  
Mira Costa High School  
Manhattan Beach, California

**Dennie Palmer Wolf**  
Senior Research Associate  
Harvard Graduate School of Education  
Cambridge, Massachusetts

**C. Kent McGuire (Ex-Officio)**  
Assistant Secretary of Education  
Office of Educational Research  
and Improvement  
U.S. Department of Education  
Washington, DC

---

**Roy Truby**  
Executive Director, NAGB  
Washington, DC

***School Policies and Practices  
Affecting Instruction  
in Mathematics:***

*Findings from the National Assessment  
of Educational Progress*

---

**Evelyn F. Hawkins  
Frances B. Stancavage**  
*American Institutes for Research*

**John A. Dossey**  
*Illinois State University*

**August 1998**

---

U.S. Department of Education  
Office of Educational Research and Improvement      NCES 98-495

Prepared by Educational Testing Service under a cooperative  
agreement with the National Center for Education Statistics

**U.S. Department of Education**  
Richard W. Riley  
Secretary

**Office of Educational Research and Improvement**  
C. Kent McGuire  
Assistant Secretary

**National Center for Education Statistics**  
Pascal D. Forgione, Jr.  
Commissioner

**Assessment Group**  
Gary W. Phillips  
Associate Commissioner

---

**August 1998**

**SUGGESTED CITATION**

Hawkins, E.F., Stancavage, F., Dossey, J.A.(1998). School Policies Affecting Instruction in Mathematics. Washington, DC: National Center for Education Statistics.

**FOR MORE INFORMATION**

Contact:  
Arnold A. Goldstein  
202-219-1741

For ordering information on this report, write:

U.S. Department of Education  
ED Pubs  
P.O. Box 1398  
Jessup, MD 20794-1398

or by calling toll free 1-877-4 ED PUBS

This report also is available on the World Wide Web: <http://nces.ed.gov/naep>.

The work upon which this publication is based was performed for the National Center for Education Statistics, Office of Educational Research and Improvement, by Educational Testing Service.

Educational Testing Service is an equal opportunity, affirmative action employer.

Educational Testing Service, ETS, and the ETS logo are registered trademarks of Educational Testing Service.

# Table of Contents

Chapter 1. Introduction .....	1
Major Findings .....	2
Who is teaching mathematics to our students? .....	3
What emphasis does mathematics instruction receive? .....	4
What are the resources in schools that support mathematics learning? .....	4
Interpreting NAEP Results .....	5
Overview of the Remainder of the Report .....	6
Chapter 2. Who is Teaching Mathematics to Our Students? .....	9
Academic Preparation .....	9
Undergraduate and graduate majors .....	9
College or university courses in mathematics .....	16
Teaching Certification .....	18
Teaching Experience .....	22
Professional Development .....	27
Knowledge of National Council of Teachers of Mathematics (NCTM) Standards .....	41
Summary .....	42
Grade 4 mathematics .....	42
Grade 8 mathematics .....	44
Chapter 3. What Emphasis Does Mathematics Instruction Receive? .....	47
Curriculum Frameworks .....	47
Graduation Requirements .....	48
Course Offerings .....	51
Instructional Time .....	56
Summary .....	62
Chapter 4. What are the Resources in Schools that Support Mathematics Learning? .....	65
Availability of Resources .....	65
Curriculum Specialists and Planning Time .....	68
Availability of Calculators .....	70
Summary .....	74
Chapter 5. Conclusion .....	77

## ***Appendices***

Appendix A	Procedural Appendix .....	A-1
Appendix B	Standard Error Tables .....	B-2

## ***Acknowledgements***

# TABLES AND FIGURES

## Chapter 2

Table 2.1	Percentage of Students and Average Scale Score by Teachers' College Major, Grades 4 and 8 .....	11
Figure 2.1	Percentage of Students Whose Teachers Have a College Major in Mathematics, for the Nation and States: Public Schools Only, 1996 .....	15
Table 2.2	Percentage of Students by Teachers' Reports on One or More College Mathematics Courses Taken, Grades 4 and 8 .....	17
Table 2.3	Percentage of Students and Average Scale Score by Teachers' Teaching Certification, Grades 4 and 8, 1996 .....	19
Figure 2.2	Percentage of Students Whose Teachers Have Mathematics Teaching Certificate, for the Nation and States: Public Schools Only, 1996 .....	21
Table 2.4	Percentage of Students and Average Scale Score by Teachers' Years of General Teaching Experience, Grades 4 and 8 .....	23
Table 2.5	Percentage of Students and Average Scale Score by Teachers' Years of Mathematics Teaching Experience, Grades 4 and 8 .....	24
Figure 2.3	Percentage of Students Whose Teachers Report More Than Ten Years of Teaching Mathematics, for the Nation and the States: Public Schools Only, 1996 .....	26
Table 2.6	Percentage of Students and Average Scale Score by Teachers' Hours of Professional Development in Mathematics or Mathematics Education During the Last Year, Grades 4 and 8, 1996 .....	28
Figure 2.4	Percentage of Grade 4 Students Whose Teachers Report More Than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996 .....	30
Figure 2.5	Percentage of Grade 8 Students Whose Teachers Report More Than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996 .....	31
Table 2.7	Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Use of Technology, Grades 4 and 8, 1996 .....	32
Figure 2.6	Percentage of Grade 4 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and States: Public Schools Only, 1996 .....	34
Figure 2.7	Percentage of Grade 8 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and the States: Public Schools Only, 1996 .....	35
Table 2.8	Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, Grades 4 and 8, 1996 .....	36



Figure 2.8	Percentage of Grade 4 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996 .....	38
Figure 2.9	Percentage of Grade 8 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996 .....	39
Table 2.9	Percentage of Students and Average Scale Score by Number of College Courses in Mathematics or Mathematics Education Teachers Have Taken During the Last Two Years, Grades 4 and 8, 1996 .....	40
Table 2.10	Percentage of Students and Average Scale Score by Teachers' Knowledge of NCTM Curriculum and Evaluation Standard, Grades 4 and 8, 1996 .....	41

### **Chapter 3**

Figure 3.1	Percentage of Twelfth-Grade Students by Mathematics Graduation Requirement (Grades 9 through 12), 1996 .....	48
Table 3.1	Percentage of Twelfth-Grade Students and Average Scale Score by Highest Algebra-Calculus Course Taken and Mathematics Graduation Requirement (Grades 9 through 12), 1996 .....	49
Table 3.2	Percentage of Twelfth-Grade Students Who Have Taken Geometry by Mathematics Graduation Requirement (Grades 9 through 12), 1996 .....	50
Table 3.3	Percentage of Twelfth-Grade Students and Average Scale Score by Whether Specific Advanced Mathematics Course of One Semester in Length Taught in Their School .....	52
Figure 3.2	Percentage of Eighth-Grade Students Whose Schools Offer Algebra for High School Credit or Placement .....	53
Figure 3.3	Percentage of Students in Schools That Offer Algebra for Eighth-Grade Students, for the Nation and States: Public Schools Only, 1996 .....	54
Table 3.4	Percentage of Eighth-Grade Students by Mathematics Course Enrollment and Availability of Algebra, 1996 .....	55
Figure 3.4	Percentage of Students Enrolled in Algebra, for the Nation and the States: Public Schools Only, 1996 .....	57
Table 3.5	Percentage of Students and Average Scale Score by Time on Mathematics Instruction, Grades 4 and 8 .....	58
Figure 3.5	Percentage of Grade 4 Students with Four or More Hours per Week of Mathematics Instruction, for the Nation and States: Public Schools Only, 1996 .....	60
Figure 3.6	Percentage of Grade 8 Students with Four or More Hours per Week of Mathematics Instruction, for the Nation and States: Public Schools Only, 1996 .....	61

## **Chapter 4**

Table 4.1	Percentage of Students and Average Scale Score by Teachers' Reports on the Availability of Resources, Grades 4 and 8 .....	66
Table 4.2	Percentage of Students and Average Scale Score by Whether Teachers Have Access to Mathematics Curriculum Specialists, Grades 4 and 8 .....	68
Table 4.3	Percentage of Students and Average Scale Score by Teachers' Designated Preparation Time per Week, Grades 4 and 8, 1996 .....	69
Table 4.4	Percentage of Student and Average Scale Score by Access to School-Owned Calculators, Grades 4 and 8 .....	71
Table 4.5	Percentage of Students and Average Scale Score by Access to Calculators for Mathematics Schoolwork, Grade 12 Enrolled in Mathematics .....	72
Table 4.6	Percentage of Students and Average Scale Score by Use of Scientific and Graphing Calculators, Grades 8 and 12, 1996 .....	73

# Chapter 1

## Introduction

The teaching and learning of mathematics in our nation's schools continue to generate tremendous attention, both among those who support recent innovations and, more recently, among those who question the wisdom of the promulgated reforms.<sup>1</sup> In order to bring an empirical basis to this debate, it is important to gather information on the policies and practices that are actually being implemented, and this report provides one source for such information. Written for policy makers and school administrators, this report is the second of a series that discusses results from the National Assessment of Educational Progress (NAEP) 1996 mathematics assessment.

General information about student performance is presented in the first report from the assessment: *NAEP 1996 Mathematics Report Card for the Nation and the States*.<sup>2</sup> A third report, tentatively titled *Student Work and Classroom Practices in Mathematics*, includes information about student performance within the various mathematics content strands; provides numerous examples of the assessment questions and student responses to those questions; and describes teachers' instructional practices and students' course-taking and attitudes towards mathematics. Finally, the fourth report in the series, tentatively titled *Focused Studies in NAEP's 1996 Mathematics Assessment*, is on special studies conducted through NAEP in 1996. These studies assessed student achievement in the following areas of mathematics: estimation, problem-solving within a real-life context, and challenging, higher-level-content problems.

In the *NAEP 1996 Mathematics Report Card for the Nation and the States*, national gains in students' mathematics scores were reported at all three grade levels: 4, 8, and 12.<sup>3</sup> Notably, average national scores, which had already increased between 1990 and 1992, increased again in 1996. Gains were also reported in many jurisdictions that took part in the NAEP state assessments. At grade 4, 15 of the 39 states and jurisdictions that participated in both the 1992

---

<sup>1</sup> Davis, R.B., Maher, C.A. & Noddings, N. (1990). *Constructivist views on the teaching and learning of mathematics*. Reston, VA: National Council of Teachers of Mathematics, Inc.; Edwards, T.G. (1994, October). *Current reform efforts in mathematics education*. *ERIC/CSMEE Digest*. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education; Jacobson, L. (1998, March 11). Experts promote math, science for preschoolers. *Education Week*, 17(26), pp.1, 12-13; Lacampagne, C.B. (1993). *State of the art: Transforming ideas for teaching and learning mathematics*. Washington, DC: Office of Research and Improvement, U.S. Department of Education; Loveless, T. (1997, October 15). The second great math revolution. *Education Week*, 17(7), pp. 48, 36; Manzo, K.K. (1997, November 5). Math showdown looms over standards in Calif. *Education Week*, 17(10), pp. 1, 18.

<sup>2</sup> Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

<sup>3</sup> Ibid.

and 1996 assessments recorded an increase in their average mathematics scores. Similar gains were evident for 13 of the 37 states and jurisdictions that participated in both the 1992 and 1996 assessments at grade 8. No state NAEP assessments were conducted at grade 12.

This report describes the educational policies and practices that prevailed during this period of sustained increases in mathematics achievement, with particular attention to the relationship between these policies and practices and student performance on the NAEP mathematics assessment. More specifically, this report provides information on the status of mathematics education in 1996 and chronicles the changes that had taken place from the time of earlier NAEP assessments.

The report is based on information provided by students and their teachers and school administrators through background questionnaires that NAEP administers concurrent with its assessments.<sup>4</sup> Students at grades 4, 8, and 12 answered questions about their home backgrounds, the instruction they received, and their course-taking. Fourth- and eighth-grade teachers who participated in the assessment provided information about their education, professional careers, curricular practices, and instructional approaches, as well as the resources available to them for teaching mathematics.<sup>5</sup> School administrators answered questions about school policies and practices.

The report is organized around three central questions:

- Who is teaching mathematics to our students?
- What emphasis does mathematics instruction receive?
- What are the resources in schools that support mathematics learning?

## **Major Findings**

The major findings reported below include information about the status of teachers and mathematics instruction in our nation's schools as well as the relationships between student achievement in mathematics and teacher characteristics and school policies and practices in mathematics education. In general, we have highlighted positive relationships. However, the reader should keep in mind the limitations of survey data of the kind collected by NAEP. Statistically significant associations between particular policies or practices and achievement can provide an interesting starting point for analysis or deliberation, but they cannot demonstrate a causal relationship. Additionally, the lack of significant changes or relationships to achievement with respect to variables reported from the NAEP survey is not necessarily evidence that our nation has remained static with regard to reforming policies and practices that positively impact mathematics education. Perhaps an appropriate conclusion from this report is that factors that

---

<sup>4</sup> For more information about all aspects of the NAEP 1996 mathematics assessment see Allen, N.A., Jenkins, F., Kulick, E., & Zelenak, C.A. (1997). *Technical report of the NAEP 1996 state assessment program in mathematics*. Washington, DC: National Center for Education Statistics.

<sup>5</sup> NAEP did not administer a survey to teachers of twelfth-grade students. In NAEP, the unit of analysis is the student and because a large proportion of twelfth-grade students — about a third — were not enrolled in mathematics classes, the information on teachers of twelfth-grade students would result in a substantial number of missing data.

impact teaching and learning of mathematics in our nation's classrooms rarely, if ever, work in isolation. The following are major findings from this report.

### **Who is teaching mathematics to our students?**

- *Teachers of the large majority of fourth-grade students (83 percent) had college majors in education rather than mathematics or mathematics education, while teachers of over half of eighth-grade students had majors in mathematics or mathematics education.*
- *Teachers' college majors appear to have some relationship to students' mathematics performance; however, there are grade-level differences. At grade 4, students whose teachers had a college major in mathematics education or education outperformed those students whose teachers had a major in a field other than education, mathematics education, or mathematics. At grade 8, it was the students of teachers with a college major in mathematics who outperformed students whose teachers had a college major in education or a field other than education, mathematics education, or mathematics.*
- *Thirteen percent of fourth-grade students were being taught mathematics by a teacher with a college major in mathematics or mathematics education. However, nearly one-third of fourth-grade students were being taught by teachers who had a mathematics teaching certificate. This pattern was also evident at the eighth-grade level: 62 percent had teachers with a college major in mathematics or mathematics education; 81 percent of eighth-grade students had teachers with a mathematics teaching certificate.*
- *Eighth-grade students whose teachers had a teaching certificate in mathematics performed better than other eighth-grade students.*
- *While teachers of fourth- and eighth-grade mathematics span the range of years of mathematics teaching experience, students taught mathematics by teachers with more than five years of teaching experience were more likely to perform better on the NAEP mathematics assessment than students taught by teachers with five or fewer years of experience.*
- *Eighth-grade teachers appear to participate in more hours of professional development in mathematics or mathematics education than fourth-grade teachers, but the level of hours of professional development was not related to students' performance in mathematics at either grade level.*
- *Teachers of eighth-grade students reported having more knowledge of NCTM curriculum and evaluation standards than teachers of fourth-grade students.*

- *The more knowledge eighth-grade teachers reported of NCTM curriculum and evaluation standards, the higher their students' performance tended to be on the NAEP mathematics assessment.*
- *Eighth-grade students enrolled in different mathematics courses — eighth-grade mathematics, pre-algebra, or algebra — appear to have had similar access to resources and opportunities to learn as evidenced by the teacher factors examined in this report.*

### **What emphasis does mathematics instruction receive?**

- *In 1996, 54 percent of our nation's twelfth-grade students were attending schools that required three or more years of mathematics in grades 9–12 for high school graduation.*
- *High school graduation requirements in mathematics appear to be related to student course-taking. Specifically, in schools that required three or more years, the percentage of students who reported having taken geometry was significantly higher than the percentage of students in schools that required two or fewer years (85 percent compared to 76 percent).*
- *Between 1990 and 1996, the percentage of eighth-grade students attending schools that offered algebra for high school credit or placement at that grade level remained stable (76–80 percent). In 1996, 28 percent of the students in such schools reported being enrolled in algebra.<sup>6</sup>*
- *Fourth-grade students were likely to receive more hours of mathematics instruction per week than eighth-grade students. In 1996, teachers of 68 percent of fourth-grade students reported that they spent 4 or more hours on mathematics instruction per week, while teachers of 33 percent of eighth-grade students reported that they spent 4 or more hours of mathematics instruction per week.*

### **What are the resources in schools that support mathematics learning?**

- *Teachers of the majority of fourth- and eighth-grade students reported that they got “most” or “all” of the instructional materials and other resources they needed to teach their class. For grade 8, higher levels of resources were found to be related to higher levels of student performance.*
- *Many teachers reported that they get “some or none” of the resources they need to teach their class (i.e., teachers of 34 percent of fourth-grade students and 21 percent of eighth-grade students).*

<sup>5</sup> As is explained in Chapter 3, algebra enrollment was also reported by 16 percent of students in other schools. Possible explanations for this finding include the following: the school offered algebra but not for automatic high school credit or placement; the student was taking algebra at the local high school or community college; or either the school or the student report was in error. Because it was not possible to verify any of these hypotheses, an assumption was made that all data were correct as reported.



- *Teachers of just less than half of fourth- and eighth-grade students reported access to a curriculum specialist to help or advise them in mathematics.*
- *Substantial proportions of students (71 percent of fourth-grade students and 88 percent of eighth-grade students) were being taught mathematics by teachers who reported having 3 or more hours per week of designated preparation time.*
- *Students' access to calculators to do schoolwork increased from 1992 to 1996. The percentage of fourth-grade students whose teachers reported that their students had access to school-owned calculators to do schoolwork increased from 59 percent in 1992 to 84 percent in 1996. Correspondingly, the percentage of twelfth-grade students taking mathematics who reported having access to calculators increased from 92 percent in 1992 to 95 percent in 1996. Data were not available for eighth-grade students in 1992, but the reported percentage with access was 80 percent in 1996.*

## **Interpreting NAEP Results**

The central mandate of NAEP is to provide information on what the nation's students know and can do in a variety of content areas. In addition, for over 25 years NAEP has regularly provided the nation's only comprehensive, recurrent data about the processes of education in the nation's schools. The latter information is intended to serve a number of important purposes. Specifically, it provides an educational context for understanding data on student achievement, it identifies differences in access to instruction and distribution of services among various types of students, and it tracks changes in policy-relevant variables across time. The findings reported above and the details in the chapters that follow are illustrations of how NAEP data serve these purposes.

However, there are some cautions that users of the information presented in this report should keep in mind. Much of the data were collected by self-report, and participants were responding to a brief, written questionnaire. Although the questions were written as clearly and unambiguously as possible, respondents working in different contexts or educated from different perspectives may have interpreted some of the questions differently. The reader should also use caution in interpreting tables that portray the association between NAEP background factors and mathematics achievement. In general, one contextual variable is presented at a time. Because of the complexity of the context in which learning takes place, examining a single variable at a time and its sole relationship to student achievement may not necessarily reveal the true underlying relationships between background factors and students' cognitive performance. For example, some instructional strategies may be used only, or most often, with high-achieving students, while other strategies may be used more frequently with lower-achieving students. Furthermore, the data reported here are cross-sectional and learning is cumulative. That is, the instructional resources examined by this report reflect a single year, not those the student has experienced in the 3, 7, or 11 years of schooling. In addition, the reader should remember that statistically significant differences may be differences that are not considered educationally significant.

Nevertheless, NAEP data are valuable, particularly when they are considered in light of other knowledge about the education system, such as trends in instructional reform, changes in the school-age population, and societal demands and expectations. Notably, they provide policy makers and administrators with a national benchmark against which to compare their own local policies and practices. Because of their basis in research, NAEP data also often help to inform our understanding of how school and instructional factors relate to achievement. Consequently, NAEP results can help practitioners to check the reasonableness of local findings in these areas. In addition, NAEP data can provide a detailed and research-based source of questions and approaches for examining local policy issues, conducting local studies, and creating local initiatives to change practice.

## **Overview of the Remainder of the Report**

This report includes four chapters and two appendices. Chapter 2 considers the academic preparation, teaching certification, years of teaching experience, and continuing professional development of teachers who provide mathematics instruction to the nation's students. The third chapter describes the emphasis that mathematics instruction receives in our schools. In particular, it examines school policies regarding curriculum, graduation requirements, mathematics courses offered, and time allotted for mathematics instruction. The fourth chapter reports on resources, including the availability of calculators, that support mathematics learning. Finally, Appendix A includes more detailed procedural information on the NAEP 1996 mathematics assessment, while Appendix B includes standard error tables for the data presented in the body of the report.

In each of Chapters 2, 3, and 4, student performance data are often presented alongside data on background variables. The average mathematics composite scale score is the indicator of student achievement used in this report.<sup>7</sup>

Mathematics reform efforts since the publication of *A Nation at Risk* have championed the notion that more students should be ready to take algebra by the eighth grade.<sup>8</sup> In this report, therefore, the eighth-grade results are disaggregated by course enrollment (algebra, pre-algebra, and eighth-grade mathematics). This allows the reader to investigate how school policies and practices differ, if they do at all, depending on the type of mathematics course in which students are enrolled.<sup>9</sup>

Information on many of the variables is also provided for public school students by state or jurisdiction, using data from the 1996 mathematics NAEP state assessment.<sup>10</sup> The NAEP 1996 mathematics assessment was conducted nationally at grades 4, 8, and 12, and state-by-state

---

<sup>7</sup> Data on student performance by mathematics content strand and NAEP achievement levels are included in other reports from the series on the NAEP 1996 mathematics assessment.

<sup>8</sup> National Commission of Excellence in Education. (1983). *A nation at risk: The imperative for educational reform*. Washington, DC: U.S. Government Printing Office.

<sup>9</sup> Although these three courses are not the only mathematics courses students reported taking, they account for the large majority of eighth-grade students. In 1996, less than half a percent of eighth-grade students indicated that they were not taking any mathematics class, about 1.5 percent reporting taking integrated (also referred to as unified or sequential) mathematics, 0.2 percent reported taking applied mathematics (also referred to as technical preparation), and 3.2 percent reported taking some other mathematics course.



at grades 4 and 8, with 44 states, the District of Columbia, Guam, the Department of Defense Domestic Dependent Elementary and Secondary Schools (DDESS), and the overseas Department of Defense Dependents Schools (DoDDS) participating. To ensure comparability across jurisdictions, NCES has established reporting guidelines related to school and student participation rates.<sup>11</sup> Results for jurisdictions failing to meet the required initial school participation rate of 70 percent are not reported, and jurisdictions failing to meet other participation guidelines are noted in the figures presenting state-by-state results.

In presenting the state-by-state data, jurisdictions are grouped by the following categories: “Percent Above the National Average,” “Percent Does Not Differ from the National Average,” and “Jurisdictions Below the National Average.” Because all results are described in terms of the percentages of students affected (e.g., percentage of students whose teachers have undergraduate or graduate major in mathematics), jurisdictions “above the national average” are those in which the percentage of students affected was significantly *higher* than the national percentage. Similarly, jurisdictions that “do not differ from the national average” are those in which the percentage of students affected was not significantly different from the national percentage, and jurisdictions “below the national average” are those in which the percentage of students was significantly *lower* than the national percentage. Also included in the cross-jurisdiction figures are 1996 average mathematics composite scale scores for all students within a given jurisdiction. That is, the score data are for all students, and not only for the particular category of students (e.g., students whose teachers majored in mathematics) being reported.

---

<sup>10</sup> At its inception in 1969, NAEP was charged with evaluating and reporting on student achievement on a national level. Subsequently, the mission of NAEP was expanded to provide state-by-state results as well. Participation in NAEP state assessment is voluntary but has grown from 40 states and territories in 1990 to 48 in 1996. Throughout this report, states and other entities participating in the NAEP state assessment are also referred to as “jurisdictions.”

<sup>11</sup> Information on these guidelines are highlighted in Reese, C.M., Miller, K.E., Mazzeo, J., & Dossey, J.A. (1997). *op. cit.*

## Chapter 2

# Who is Teaching Mathematics to Our Students?

The teaching profession has been referred to as the foundation of elementary and secondary education, touching our children most directly in the persona of their classroom teachers.<sup>1</sup> Past and current reform efforts have focused on strengthening the qualifications of these professionals via teacher education, licensing and certification, and professional development. NAEP, which has regularly surveyed teachers of students participating in its assessments, has tracked the penetration of these efforts. For the 1996 NAEP mathematics assessment, questions to teachers focused on those experiences believed — or shown in past research — to have had some influence on the teaching of mathematics and, consequently, on students' mathematics achievement.<sup>2</sup> This chapter presents the 1996 survey results on who is teaching mathematics. Where comparable data are available, information is also presented from 1992 and 1990.

### Academic Preparation

#### Undergraduate and graduate majors

In 1952, nearly half of the nation's 600,000 public elementary-school teachers did not hold college degrees.<sup>3</sup> By the early 1990s, all states required that teachers have an undergraduate degree to receive a teaching certificate, with some states requiring an additional year of study.<sup>4</sup> Those who planned to teach at the elementary-school level typically took a major in education with only a modest amount of subject-specific coursework beyond their institution's general education requirements. Those who planned to teach at the secondary level, by contrast, tended to major in the academic discipline in which they planned to teach, and they took only a few education courses.

<sup>1</sup> Darling-Hammond, L. (1996). What matters most: A competent teacher for every child. *Phi Delta Kappan*, 78(3), pp. 193-200.

<sup>2</sup> Hanushek, E.A. (1997). Assessing the effects of school resources on student performance: An update. *Educational Evaluation and Policy Analysis*, 19(2), 141-164; Spillane, J.P. & Thompson, C.L. (1997). Reconstructing conceptions of local capacity: The local education agency's capacity for ambitious instructional reform. *Educational Evaluation and Policy Analysis*, 19(2), 185-203; National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

<sup>3</sup> Lucas, C. (1997). *Teacher Education in America*. New York: St. Martin's Press.

<sup>4</sup> Henke, R.R., Choy, S.P., Chen, X., Geis, S., Alt, M.N., & Broughman, S.P. (1997). *America's Teachers: Profile of a Profession, 1993-94*. Washington, DC: National Center for Education Statistics.

Current proponents of educational reform have advocated that potential teachers of elementary-school students pay greater attention to disciplinary content and that potential teachers of secondary-level students take more education coursework. There has been substantial debate surrounding the issue of whether teachers should be allowed to major in education at the undergraduate level at all, or whether they should be required to obtain an undergraduate degree in another field and undertake the study of education at the graduate level. The Carnegie Forum and the Holmes Group issued widely-acknowledged reports that recommended teachers be required to earn a bachelor's degree in arts or science rather than in education.<sup>5</sup> This was an attempt to ensure that teachers leave school with a thorough grounding in the subjects they plan to teach. It did not, however, speak to the concerns of elementary-level teachers, the large majority of whom teach self-contained classrooms and are, therefore, expected to cover all core subjects. In fact, there is disagreement within the education community about what prospective elementary school teachers *should* be required to undertake with regard to subject matter preparation.<sup>6</sup>

NAEP survey data provide an opportunity to investigate the academic backgrounds of teachers who taught mathematics to a nationally representative sample of elementary- and middle-school students in the mid-1990s.<sup>7</sup> Table 2.1 presents the responses to the first of several relevant survey questions, showing the distributions of college majors among fourth- and eighth-grade teachers of mathematics surveyed in 1996 and 1992.<sup>8</sup> Based on teachers' responses, academic majors were categorized into one of four mutually exclusive categories:

1. *Mathematics* includes teachers with an undergraduate or graduate major in mathematics.
2. *Mathematics education* includes teachers with an undergraduate or graduate major in mathematics education, but not in mathematics.<sup>9</sup>
3. *Education* includes teachers with an undergraduate or graduate major in education, elementary education, or secondary education, but not mathematics or mathematics education.
4. *Other* includes teachers who responded with any majors other than the above.

<sup>5</sup> Carnegie Forum on Education and the Economy, Task Force on Teaching as a Profession. (1986). *A nation prepared: Teachers for the twenty-first century*. New York: Author ; Holmes Group. (1986). *Tomorrow's teachers: A report of the Holmes Group*. East Lansing, MI: Michigan State University.

<sup>6</sup> Lucas C. (1997). op. cit.

<sup>7</sup> NAEP did not select a random sample of teachers of mathematics, but rather surveyed teachers of participating students.

<sup>8</sup> The term "teachers of mathematics" refers to the teachers who were providing mathematics instruction for the fourth- and eighth-grade students participating in the assessment. Many of these teachers, therefore, particularly at the fourth-grade level, were generalists who taught all or most core subjects.

<sup>9</sup> There are differences between the programs of mathematics education majors and mathematics majors. Mathematics education degree programs typically include coursework in mathematics, methods of teaching mathematics, generic education courses, and 8–10 semester hours of student teaching. The mathematics in such programs consists of coursework directly related to teaching secondary school mathematics: calculus, linear algebra, college geometry, discrete mathematics, abstract algebra, and probability and statistics. By contrast, mathematics degree programs usually include more and different courses in mathematics. For example, students pursuing a mathematics major may have courses in differential equations, complex variables, numerical analysis, or other applied areas, but they will probably not have any courses related to the teaching of mathematics or general educational principles and practices.

Table 2.1

**Percentage of Students and Average Scale Score by Teachers' College Major, Grades 4 and 8**



	Undergraduate or Graduate Major								
		Mathematics		Mathematics Education but not Mathematics		Education but not Mathematics or Mathematics Education		Other	
	Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4									
All Students	1996	9	220	4	235	83	225†	4	208
	1992	7	221	4	229	86	220	3	215
Grade 8									
All Students	1996	49	278	13	270	32	269	7†	267
	1992	51	274	12	271	26	263	11	262
Students Enrolled in:									
Eighth-Grade Mathematics	1996	46	267†	13	259	35	261	6†	253
	1992	49	260	13	257	27	254	12	250
Pre-Algebra	1996	48	272	16	267	28	271	7	271
	1992	50	275	10	272	29	270	11	270
Algebra	1996	54	300	10	297	30	292	6	286
	1992	59	302	14	302	18	293	9	294

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

In 1996, 83 percent of fourth-grade students were being taught mathematics by teachers who had an undergraduate or graduate major in education. In contrast, nine percent of fourth-grade students were being taught mathematics by teachers who had an undergraduate or graduate major in mathematics. An additional four percent of students were taught by teachers

who had an undergraduate or graduate major in mathematics education. The high percentage of students whose teachers had a major in education is perhaps not surprising, because according to principals' reports, 53 percent of these fourth-grade students were in schools where fourth-grade students were organized into self-contained classrooms.<sup>10</sup> This means that their teachers were expected to teach all subjects, not just mathematics. Another 41 percent of fourth-grade students were reported to be in schools where they remained with one teacher for most subjects but might have a different teacher for one or two subjects. The survey did not specify which subjects were assigned to different teachers, but there is little evidence in the data to suggest that mathematics instruction was being separated out and assigned to a mathematics specialist.

Although conventional wisdom calls for teachers of mathematics to have a strong academic grounding in mathematics, fourth-grade students who were taught by teachers with an undergraduate or graduate major in mathematics or mathematics education did not perform better on the 1996 mathematics assessment than students whose teachers had an undergraduate or graduate major in education. However, students who had teachers with an undergraduate or graduate major in either mathematics education or education outperformed students whose teachers had a major in an "other" field.

The distribution of majors among the mathematics teachers of fourth-grade students did not change significantly between 1992 and 1996. That is, the percentage of students being taught mathematics by teachers with an undergraduate or graduate major in mathematics in 1996 was similar to the percentage in 1992. This was also true of teachers with majors in mathematics education, education, or "other" fields.

In contrast to fourth grade where the majority of students were in self-contained classrooms, 87 percent of eighth-grade students assessed in 1996 were in schools where eighth grades were organized departmentally.<sup>11</sup> Given this organizational pattern of specialization, one would more reasonably expect eighth-grade students to have been taught mathematics by teachers with an undergraduate or graduate major in mathematics or mathematics education. In fact, 49 percent of students had teachers with a mathematics major, while another 13 percent had teachers with a major in mathematics education. Nearly 40 percent of eighth-grade students, however, were in mathematics classes taught by teachers with a major in education or an "other" field. This preparation may not be optimal given the level of mathematics content that many now consider appropriate for eighth-grade students.<sup>12</sup> Student performance data show that, unlike at the fourth-grade level, eighth-grade students taught by teachers with a major in mathematics outperformed students taught by teachers with a major in education or an "other" field.

Given that 62 percent of eighth-grade students were being taught mathematics by teachers with a major in mathematics or mathematics education, the question arises as to whether students in more advanced mathematics courses were more likely to be taught by teachers with these majors. The data show that this was not the case. For example, in 1996, the percentages of

---

<sup>10</sup> The source of the data on classroom organizational structure is the NAEP 1996 mathematics assessment.

<sup>11</sup> The source of these data is the NAEP 1996 mathematics assessment.

<sup>12</sup> Mathematical Sciences Education Board. (1997). *Toward Excellence in K-8 Mathematics*. <<http://www2.nas.edu/mseb/213e.html>>; U.S. Department of Education, Office of the Under Secretary, Planning and Evaluation Service, Partnership for Family Involvement in Education. (1997). *Getting Ready for College Early: A Handbook for Parents of Students in the Middle and Junior High School Years*. <<http://www.ed.gov/pubs/GettingReadyCollegeEarly/>>.

eighth-grade mathematics, pre-algebra, and algebra students taught mathematics by teachers with a mathematics major were not significantly different from each other. This was also true of the percentages of students with teachers with a mathematics education, education, or “other” major.

As with fourth-grade students, between 1992 and 1996, the percentages of eighth-grade students taught by teachers in the various categories of majors did not change significantly except for the “other” major category. The percentage of students taught mathematics by teachers with an “other” major, decreased significantly from 11 percent in 1992 to 7 percent in 1996.

Over the years, states have responded differently to the national debate regarding teacher education. Although policies about college degrees and academic major requirements are generally the purview of the granting institutions of higher education, their policies are naturally guided, and sometimes controlled, by states' licensure and certification requirements. A few states have enacted new policies regarding academic requirements for teacher licensure and certification. In response, many teacher-training programs have moved or have considered moving away from undergraduate degrees in general education to teacher education programs in which students are required to combine an undergraduate degree in a specific discipline with additional coursework in education-related areas that leads to a graduate degree in education or teaching.

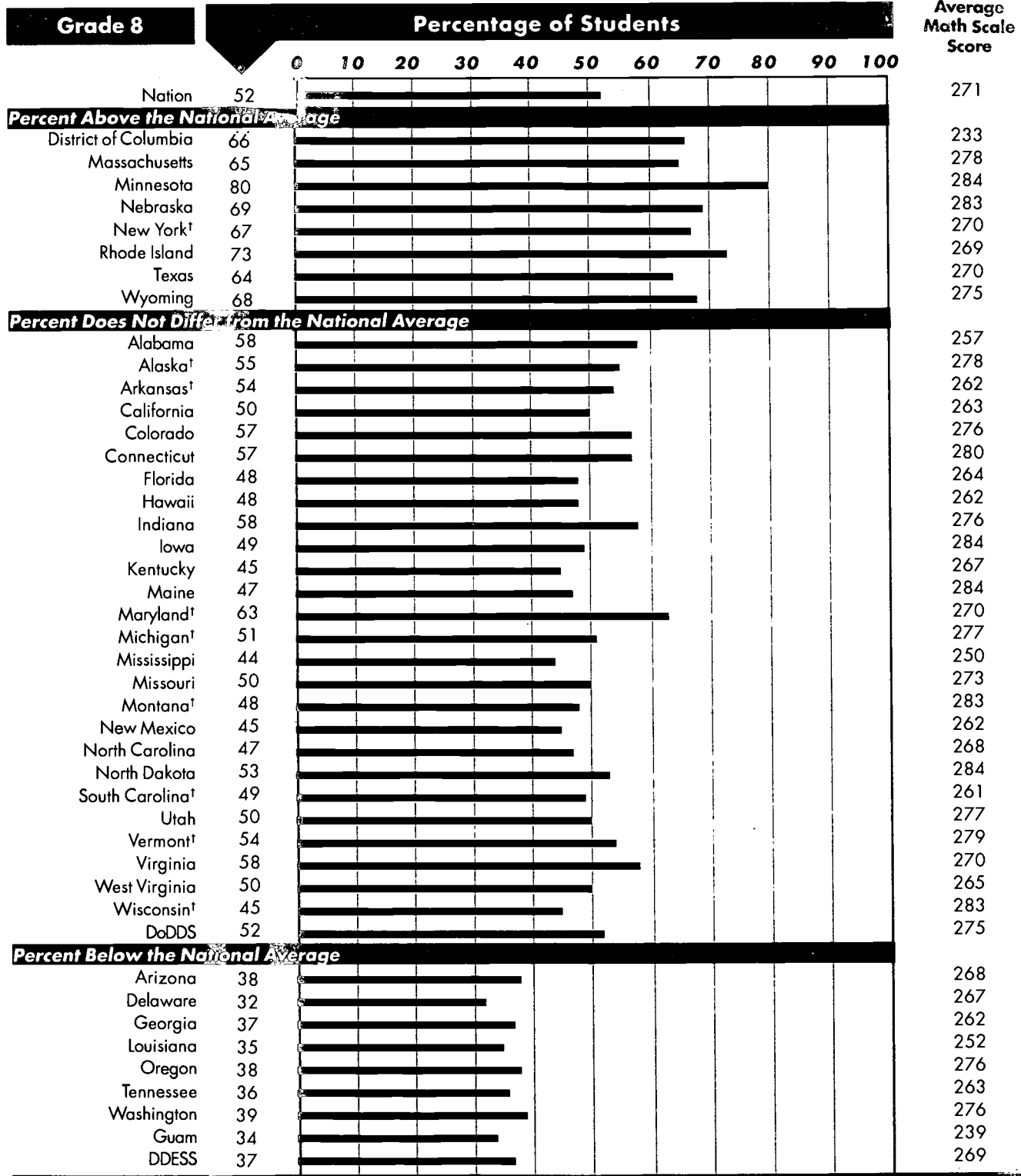
A study by the Council of Chief State School Officers (CCSSO) on state education policies in 1996 reported that 10 states had an explicit requirement that all new teachers hold a major in a specific subject field, and 22 additional states required a major in a specific field for new secondary teachers.<sup>13</sup> Figure 2.1 presents state information on the percentage of eighth-grade students who, in 1996, were taught mathematics by teachers who reported an undergraduate or graduate major in mathematics. There were eight jurisdictions in which the percentage of eighth-grade students who had teachers with an undergraduate or graduate major in mathematics was significantly beyond the national average, 27 jurisdictions in which the percentages were near the national average, and a remaining nine jurisdictions in which the percentages were less than the national average.

<sup>13</sup> Council of Chief State School Officers. (1996). *Key state education policies on K-12 education: Content standards, graduation, teacher licensure, time and attendance*. Washington, DC: Author.



Figure 2.1

**Percentage of Students Whose Teachers Have a College Major in Mathematics, for the Nation and States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.



### **College or university courses in mathematics**

Although an academic major is generally considered to be advantageous in preparing teachers to present disciplinary content more effectively to students, teachers — even teachers employed in high schools or middle schools — are often required to teach more than one subject as part of their regular assignment.<sup>14</sup> Consequently, to require teachers to have a major in each field that they expect to teach may be less than reasonable. However, teachers may take mathematics courses regardless of whether or not they major in mathematics or mathematics education.

Teachers of students participating in the NAEP assessment were asked to indicate their level of exposure to college or university courses in various areas of mathematics and mathematics pedagogy, including the following: methods of teaching elementary mathematics, number systems and numeration, measurement in mathematics, geometry, probability or statistics, college algebra, calculus, and abstract/linear algebra. Table 2.2 shows the percentages of fourth- and eighth-grade students whose teachers indicated taking at least one college course in the specified content area.

The data show that, in 1996, a high percentage of fourth-grade students (84%) were being taught mathematics by teachers who had taken one or more college courses in methods of teaching elementary mathematics. One half or less of fourth-grade students, however, had teachers who had been exposed to one or more college courses in any of the other mathematics content areas. Comparisons to 1992 do not suggest progress on this indicator of subject matter knowledge. In 1996, the percentages of fourth-grade students whose teachers reported having taken college-level courses in each of the surveyed content areas were either stable or declining.

At the eighth-grade level, across the various content areas, the 1996 percentages are somewhat more encouraging; however, this would be expected because many more teachers of eighth-grade students had majors in mathematics or mathematics education. There was only one content area in mathematics in which a smaller percentage of eighth-grade students compared to fourth-grade students had teachers with one or more college courses: namely, methods of teaching elementary mathematics. This would be expected because only 44 percent of eighth-grade students, while 84 percent of fourth-grade students, had teachers with this background. In addition, the percentages of fourth- and eighth-grade students whose teachers had one or more college courses in number systems and numeration and in measurement in mathematics were not significantly different from each other.

In 1996, eighth-grade students enrolled in different types of mathematics courses were being taught mathematics by teachers with similar levels of exposure to college courses in the different mathematics content areas. The proportions of students enrolled in algebra whose teachers had taken courses in the various content areas were about the same as the proportions of students enrolled in pre-algebra or eighth-grade mathematics.

Comparisons with 1992 percentages show that, in 1996, smaller percentages of eighth-grade students were being taught mathematics by teachers who had been exposed to courses in methods of teaching elementary mathematics or abstract or linear algebra. The reader should use caution in interpreting the latter difference, however, because the 1992 NAEP

---

<sup>14</sup> Lucas, C. (1997). op. cit.; National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

questionnaire did not include college algebra as an option. Exposure to coursework in college algebra was widely reported by the 1996 respondents (almost three-fourths of eighth-grade students in 1996 had teachers who indicated such coursework), and it is possible that, in the absence of the college algebra response option, some 1992 teachers marked abstract or linear algebra who would not otherwise have done so.

The reader also is cautioned about assumptions regarding the kind of mathematics preparation provided by college algebra courses. This is because “college algebra” may have been interpreted by some teachers as the pre-calculus class by that name, offered in many colleges, while others may have interpreted “college algebra” as a post-calculus course in linear

**Table 2.2**

**Percentage of Students by Teachers' Reports on One or More College Mathematics Courses Taken, Grades 4 and 8**



	Mathematics Course Content Area								
	Assessment Year	Teaching Methods	Number Systems and Numeration	Measurement	Geometry	College Algebra	Probability/ Statistics	Calculus	Abstract/ Linear Algebra
Grade 4									
All Students	1996	84†	43†	37	34	45	36†	13	13†
	1992	92	53	43	33	NA	45	13	31
Grade 8									
All Students	1996	44†	50	37	64	74	68	68	54†
	1992	58	56	47	66	NA	67	66	67
Students Enrolled in:									
Eighth-Grade Mathematics	1996	47	49	35	62	74	67	62	52†
	1992	55	54	45	66	NA	68	67	67
Pre-Algebra	1996	42†	49	36	65	74	66	69	53
	1992	58	59	48	62	NA	66	61	64
Algebra	1996	40†	53	42	69	75	73	77	62†
	1992	65	60	50	72	NA	71	75	76

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

or abstract algebra.<sup>15</sup> Nevertheless, the information presented in Table 2.2 provides some indication of the level of preparation of teachers in the different mathematics content areas.

## **Teaching Certification**

The type of teaching certificate that teachers hold is typically dependent upon their college degrees and college coursework. However, state policies can also affect the types of teaching certificates teachers at different grade levels hold; for example, in some states certificates are not available in elementary mathematics, while in other states they are. With this caveat in mind, Table 2.3 presents information about the types of teaching certificates teachers held. For the purposes of this report, teachers were categorized into three mutually exclusive certification categories:

1. *Mathematics* which includes teaching certificates in elementary mathematics or in middle/junior high or secondary mathematics.
2. *Education* which includes teaching certificates in elementary or middle/junior high education but not in elementary mathematics or middle/junior high or secondary mathematics.
3. *Other* which includes teaching certificates in fields other than those included in the Mathematics or Education categories.

Two-thirds of fourth-grade students were in mathematics classes taught by teachers who had teaching certificates in education, while nearly all of the remaining students were taught by teachers with certificates in mathematics. The types of teaching certificates held by their teachers appears not to be related to fourth-grade students' performance on the 1996 NAEP mathematics assessment.

Perhaps as expected, the large majority of eighth-grade students, 81 percent, were in mathematics classes taught by teachers with teaching certificates in mathematics. In contrast, 18 percent had mathematics teachers with teaching certificates in education, and only 2 percent had teachers with teaching certificates in an "other" field. At the eighth-grade level, students who were taught by teachers with teaching certificates in mathematics outperformed students whose teachers had teaching certificates in education or an "other" field; students whose teachers had teaching certificates in mathematics had an average mathematics scale score of 276, while those students whose teachers had teaching certificates in education had an average scale score of 265, and those students whose teachers had teaching certificates in an "other" field had an average scale score of 248.

---

<sup>15</sup> Discussion with NAEP Mathematics/Science Standing Committee, October 1997.

Table 2.3

**Percentage of Students and Average Scale Score by  
Teachers' Teaching Certification,  
Grades 4 and 8, 1996**



	Type of Teaching Certification					
	Mathematics		Education but not Mathematics		Other	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>						
All Students	32	225	67	224	11	223
<b>Grade 8:</b>						
All Students	81	276	18	265	21	248
<b>Students Enrolled in:</b>						
Eighth-Grade Mathematics	78	266	20	258	21	240
Pre-Algebra	82	272	16	270	11	***
Algebra	86	299	13	289	11	***

\*\*\*Sample size is insufficient to permit a reliable estimate.

1 Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Regardless of the mathematics course in which they were enrolled, the large majority of eighth-grade students were being taught mathematics by teachers who held mathematics teaching certificates. Specifically, 22 percent of students enrolled in eighth-grade mathematics, 19 percent of students enrolled in pre-algebra, and 14 percent of students enrolled in algebra had teachers without mathematics teaching certificates.

In the early 1900s, teaching certificates specific to grade levels — elementary, middle, or secondary — or to a particular disciplinary content area were uncommon.<sup>16</sup> By 1930, however, nearly all states issued certificates specifically for teaching in the elementary grades; 26 states issued licenses for junior high school teachers; and 31 states issued them for high school teachers. Now, reform efforts in mathematics education advocate hiring teachers with teaching certificates in mathematics to teach mathematics. This is already the norm at the secondary level, with reform efforts it is becoming increasingly more widespread at the middle school level.<sup>17</sup> Figure 2.2 presents 1996 state data on the percentage of eighth-grade students whose teachers report having teaching certificates in mathematics. In 1996, 18 jurisdictions had percentages of students with teachers holding mathematics teaching certificates that were significantly higher than the national percentage, 15 jurisdictions had percentages of students that were similar to the national percentage, and 11 jurisdictions had percentages of students that were less than the national percentage.

The number of teachers of mathematics with teaching certification in mathematics, and consequently, the percentage of students taught mathematics by teachers with a mathematics teaching certification in a state, district, or school is influenced by many factors. These factors range from hiring policies, to the supply of teachers with mathematics teaching certificates, to the academic needs of students, and to the perceived impact of teachers with mathematics teaching certificates on meeting those needs.

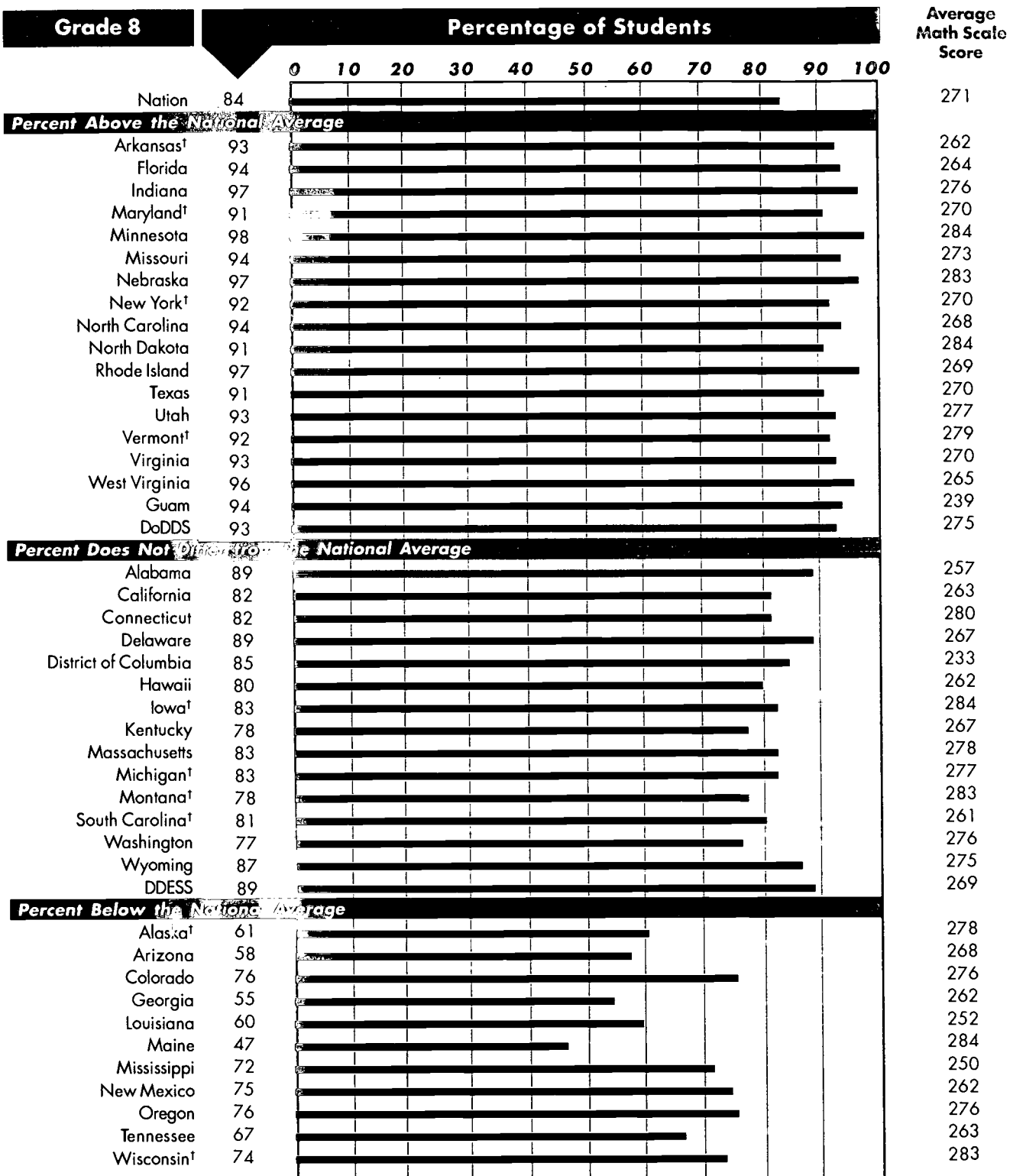
---

<sup>16</sup> Lucas. C. (1997). op. cit.

<sup>17</sup> Mathematical Sciences Education Board. (1997). op. cit.

Figure 2.2

**Percentage of Students Whose Teachers Have  
Mathematics Teaching Certificate, for the Nation and  
States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

## Teaching Experience

Teaching experience can be viewed as a resource to which students have access. It is therefore instructive to examine the duration of teaching experience, both in general and for mathematics in particular. However, the reader must bear in mind that *extent* of teaching experience is not synonymous with *quality* of teaching experience. On the one hand, teachers with more teaching experience might have worked with a greater diversity of students and consequently developed a greater repertoire of instructional strategies. On the other hand, teachers who have been in the workforce longer might have more dated information about mathematics education unless they continue to engage in professional development activities. The data in Tables 2.4 and 2.5 provide information about the amount of teaching experience — in general and in mathematics, specifically — possessed by teachers who, in 1996 and 1992, were teaching mathematics to fourth-grade and eighth-grade students.

Table 2.4 considers *general* elementary or secondary teaching experience regardless of subjects taught. In 1996, the modal amount of general teaching experience for teachers of fourth-grade mathematics was 11–24 years. Students who were taught by teachers with less than 5 years of teaching experience had performance below the performance of students whose teachers had 6–10 years or 25 or more years of teaching experience.

In examining trends over time, there is some evidence that the 1996 teaching workforce for fourth-grade mathematics had slightly fewer years of teaching experience than in 1992. That is, although the highest percentages of students in both 1996 and 1992 were taught by teachers with experience in the 11–24 year range, the percentage of students in this category fell from 46 percent in 1992 to 35 percent in 1996. At the same time, the percentage of students taught by teachers with 6–10 years of teaching experience increased from 15 percent in 1992 to 24 percent in 1996.

In 1996, 38 percent of eighth-grade students were taught mathematics by teachers who had 11–24 years of teaching experience. Students whose teachers had 11–24 years of teaching experience outperformed students whose teachers had 5 or fewer years of general teaching. The 1996 percentages of students whose teachers had a given level of years of teaching experience were not significantly different from the 1992 percentages.

Table 2.4

**Percentage of Students and Average Scale Score by Teachers' Years of General Teaching Experience, Grades 4 and 8**



		Years of Elementary or Secondary Teaching Experience							
		5 Years or Less		6-10 Years		11-24 Years		25 Years or More	
	Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
All Students	1996	22	219	24†	226	35†	224	19	228
	1992	21	215	15	218	46	222	18	222
<b>Grade 8</b>									
All Students	1996	20	268	19	274	38	275†	24	274
	1992	20	265	15	266	47	270	17	276
<b>Students Enrolled in:</b>									
Eighth-Grade Mathematics	1996	20	260	22	266†	39	265	20	260
	1992	22	253	15	254	47	257	16	262
Pre-Algebra	1996	24	269	18	272	35†	273	24	269
	1992	18	269	16	263	51	276	15	278
Algebra	1996	16	289	15	302	40	296	29	300
	1992	19	297	13	303	45	298	22	304

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.



Information on teachers with more specialized teaching experience, that is, experience in teaching mathematics, is presented in Table 2.5. Trend data are only presented for the teachers of eighth-grade students because, in 1992, this question was not asked of teachers of fourth-grade students.

**Table 2.5**

**Percentage of Students and Average Scale Score by Teachers' Years of Mathematics Teaching Experience, Grades 4 and 8**



		Years of Mathematics Teaching Experience							
		5 Years or Less		6-10 Years		11-24 Years		25 Years or More	
	Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
All Students	1996	26	219	26	227	33	224	15	229
<b>Grade 8</b>									
All Students	1996	27	269	20	272	37	276	17	277
	1992	26	264	18	267	43	271	13	276
<b>Students Enrolled in:</b>									
Eighth-Grade Mathematics	1996	28	262†	20	262	38	265	13	264
	1992	28	253	16	254	43	259	13	263
Pre-Algebra	1996	29	269	20	273	34	273	17	269
	1992	25	268	21	267	43	278	11	278
Algebra	1996	21	290	19	295	40	298	21	302
	1992	22	297	18	301	44	299	16	304

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

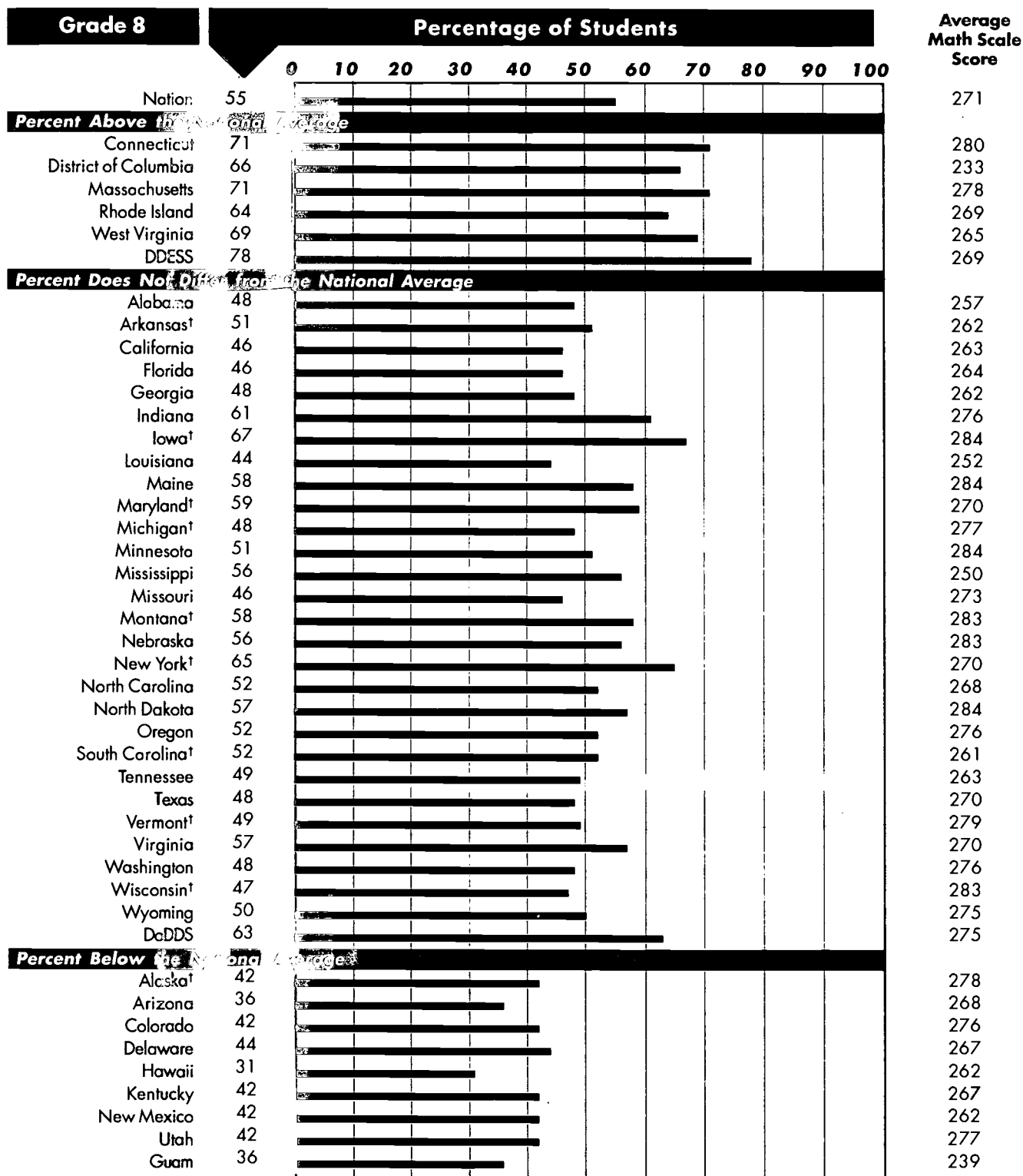
A comparison of Tables 2.4 and 2.5 shows that, for fourth-grade students, teachers tended to report about the same amount of mathematics teaching experience than general teaching experience; however, none of the apparent differences were statistically significant. This is reasonable given that most fourth-grade teachers are teaching a general curriculum in self-contained classrooms. Moreover, whether the measure is general teaching experience or mathematics teaching experience, fourth-grade students taught by teachers with 6–10 years or 25 or more years of experience performed better on the 1996 NAEP mathematics assessment than students taught by teachers with 5 or fewer years of experience.

At the eighth-grade level, the relationship of lower student performance to teachers with 5 or fewer years of experience continues to hold; that is, these students performed lower in comparison to students whose teachers had 11–24 years of mathematics teaching experience. However, within course type, no significant relationship between years of mathematics teaching experience and student performance was observed. The percentages of eighth-grade students being taught by teachers with a given level of mathematics teaching experience did not change significantly between 1992 and 1996.

Mathematics continues to be one of the more important subjects in the school core curriculum. Consequently, districts and schools, when their budgets allow, are likely to seek to hire teachers, whether at the fourth- or eighth-grade level, who have more years of experience teaching mathematics. At the same time, both general and specialized years of experience tends to vary with the overall age of the teaching workforce in different areas. These trends play out differently in different states. Figure 2.3 presents state information on the percentages of eighth-grade students taught mathematics by teachers with more than 10 years of experience teaching mathematics. The majority of jurisdictions had percentages of students being taught mathematics by teachers who reported more than 10 years of experience teaching mathematics that were similar to the national percentage, six had percentages that were higher, and nine had percentages that were lower than the national percentage.

Figure 2.3

**Percentage of Students Whose Teachers Report More Than Ten Years of Teaching Mathematics, for the Nation and the States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

## **Professional Development**

The mathematics reform movement has provided an impetus for more professional development opportunities for teachers. Mathematics teaching methods being advocated currently are substantially different from methods utilized a decade ago.<sup>18</sup> In addition, despite the recent reform efforts, many schools of education are still adhering to curricula that do not reflect currently advocated methods of teaching mathematics.<sup>19</sup> To accommodate the needs of teachers who are being asked to make substantial changes in the curricula they employ and the instructional strategies they use, efforts are being made to provide more and higher-quality professional development opportunities. In addition to the college or university offerings that teachers have historically accessed, national, state, and local efforts have multiplied the opportunities for teachers' professional development.<sup>20</sup>

Goal 4 of the Goals 2000 Act reflects the belief that teachers should have access to high-quality professional development throughout their careers.

*By the year 2000, the Nation's teaching force will have access to programs for the continued improvement of their professional skills and the opportunity to acquire the knowledge and skills needed to instruct and prepare all American students for the next century.*<sup>21</sup>

Data about teachers' participation in professional development activities are presented in Table 2.6. They are based on a question on the NAEP teacher questionnaire that specifically referenced such professional development activities as workshops and seminars, attendance at professional meetings and conferences, district-sponsored (internal) workshops, and external workshops.

Despite efforts to increase professional development opportunities for teachers, in 1996, 46 percent of fourth-grade students were taught by teachers who indicated having participated in less than six hours of professional development in mathematics or mathematics education during the preceding year, 27 percent had teachers who indicated they participated in 6–15 hours of professional development, and 27 percent were taught by teachers who indicated they participated in more than 15 hours of professional development. Of course, these findings do not necessarily mean that fourth-grade teachers did not participate in many more hours of professional development in other topics, including those that cut across disciplinary content areas and may have helped to improve their teaching of mathematics. Student performance on the 1996 NAEP mathematics assessment was not found to be related to the hours of professional development in mathematics teachers received.

---

<sup>18</sup> National Council of Teachers of Mathematics. (1991). op. cit.

<sup>19</sup> Wise, A. E. (1996). Building a system of quality assurance for the teaching profession. *Phi Delta Kappan*, 78(3), pp. 190-192; Bradley, A., (1995). Holmes group urges overhaul of ed. schools. *Teacher Magazine*. <<http://www.teachermag.org/ew/vol-14/19holm.h14>>.

<sup>20</sup> Henke, Choy, Chen, Geis, Alt. & Broughman. (1997). op. cit.

<sup>21</sup> *Goals 2000: Educate America Act*. (1994).

Table 2.6

**Percentage of Students and Average Scale Score by Teachers' Hours of Professional Development in Mathematics or Mathematics Education During the Last Year, Grades 4 and 8, 1996**



		Hours of Professional Development					
		Less than 6 Hours		Between 6 and 15 Hours		More than 15 Hours	
<b>Grade 4:</b>							
	All Students	46	225	27	222	27	225
<b>Grade 8:</b>							
	All Students	26	275	29	274	45	272
<b>Students Enrolled in:</b>							
	Eighth-Grade Mathematics	25	264	28	263	47	263
	Pre-Algebra	28	270	30	273	43	270
	Algebra	28	299	29	295	43	297

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

In contrast to fourth-grade students, 45 percent of eighth-grade students were taught by teachers who reported participating in more than 15 hours of professional development in mathematics or mathematics education. This percentage is more encouraging. However, the question asked of teachers does not differentiate between 15 hours and higher levels, for example, 50 hours, of professional development in a year. The reader should bear in mind that 15 hours is still less than two 8-hour days of professional development per year. At this grade level also, student performance was found not to be related to hours of professional development received by teachers.

The data appear to indicate that the type of mathematics course students were taking was not related to the amount of professional development their teachers received. For example, the percentages of students taking eighth-grade mathematics, pre-algebra, and algebra being taught mathematics by teachers who participated in more than 15 hours of professional development did not differ significantly from each other.

BEST COPY AVAILABLE

37

One of the indicators for measuring states' progress in meeting Goals 2000 objectives is whether the percentage of a state's teachers participating in any type of professional development increased from one year to the next.<sup>22</sup> Current research also calls for participation in high-quality professional development that is of sufficient intensity, and sustained over time, although these aspects of professional development were not assessed by NAEP in 1996.<sup>23</sup> Figures 2.4 and 2.5 present state information on the percentage of students taught mathematics by teachers with more than 15 hours of professional development in mathematics or mathematics education during the past year.

At both the fourth- and eighth-grade levels, the large majority of jurisdictions had percentages of students whose teachers reported more than 15 hours of professional development that were similar to the national average. A comparison of grade 4 and grade 8 jurisdiction ranking in relation to the national percentage shows similarities and differences. For example, at both the fourth- and eighth-grade levels, California, Massachusetts, and Texas had percentages of students "above the national average" while Indiana, Wyoming, and Guam had percentages "below the national average." Arkansas and Vermont had percentages "above the national average" at the fourth-grade level, but at the eighth-grade level these states had percentages that "did not differ from the national average." The District of Columbia, Florida, and Kentucky had percentages "above the national average" at the eighth-grade level while at the fourth-grade level these jurisdictions had percentages that "did not differ from the national average."<sup>24</sup>

The data appear to provide evidence that state and local educational agencies, including schools, have different professional development policies. This is due to differing philosophies of states, school boards, and district and school administrations, and differences in the identified needs of students and teachers. In addition, differences in the data also reflect teachers' decisions about whether or not to take advantage of opportunities offered to them.

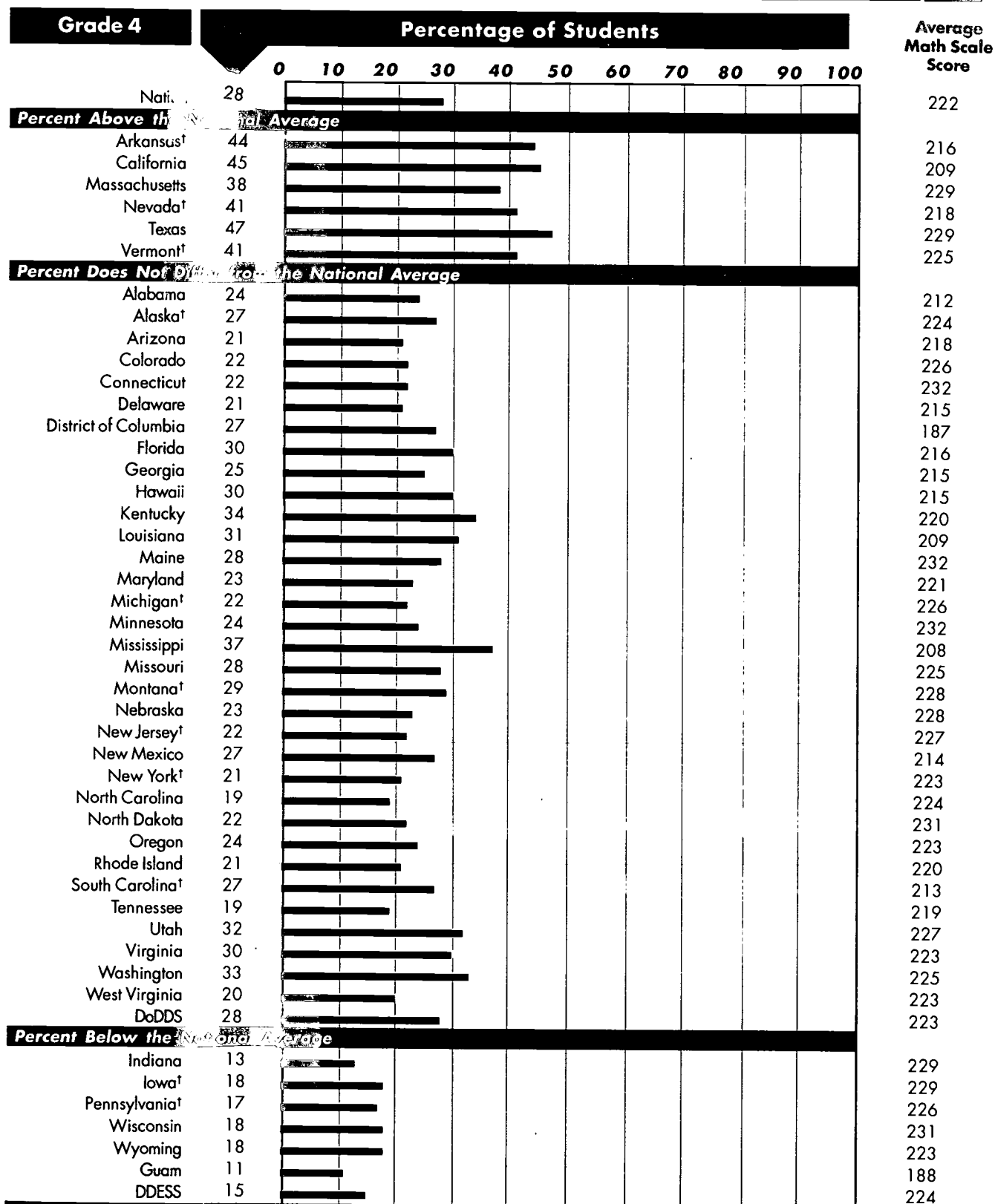
<sup>22</sup> National Education Goals Panel. (1997). *The National Education Goals Report: Building a Nation of Learners, 1997*. Washington, DC: U.S. Government Printing Office.

<sup>23</sup> Ball, D.K. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan*, 11(7), 500-508; Henke, Choy, Chen, Geis, Alt, & Broughman (1997). op. cit.; Sparks, D. (1995). Focusing staff development in improving student learning. *Handbook of Research of Improving Student Achievement*. Arlington, VA: Educational Research Service, pp. 163-169.

<sup>24</sup> In this and other state-by-state figures, jurisdictions with the same percentages may fall into different categories, for example, the District of Columbia and Mississippi in Figure 2.5. Furthermore, jurisdictions with a higher percentage such as Rhode Island may fall in a category below the category in which a jurisdiction with a lower percentage, such as Tennessee, is listed. This is because of the differing magnitudes of the standard errors of the percentages.

Figure 2.4

**Percentage of Grade 4 Students Whose Teachers Report More Than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996**



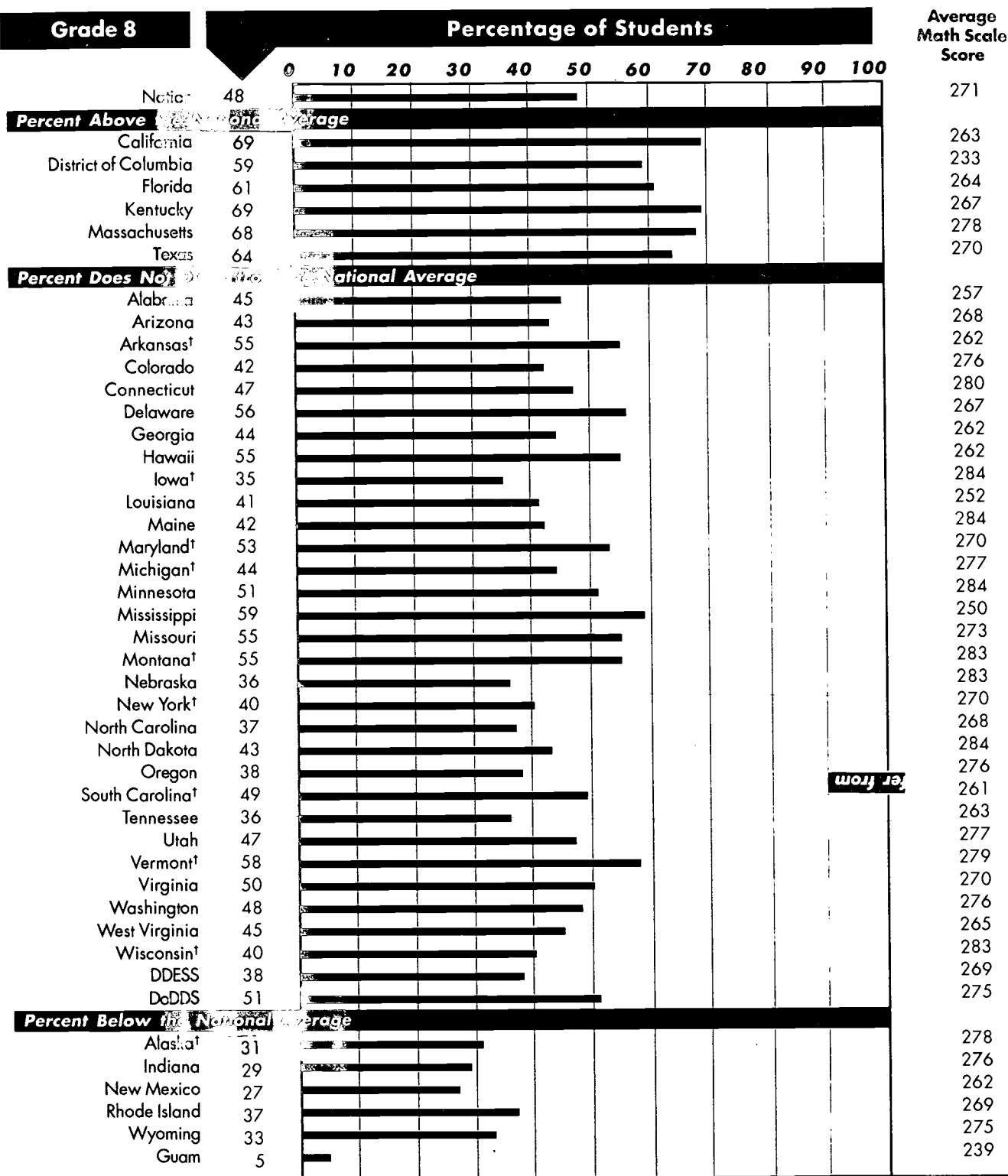
† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.



Figure 2.5

**Percentage of Grade 8 Students Whose Teachers Report More Than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.



Professional development activities offered for teachers cover a variety of content. The NAEP teacher questionnaire asked teachers to indicate whether during the past five years they had taken a course or participated in professional development activities covering certain specific pedagogical and instructional practices. Among these practices were “the use of technology such as computers” and “teaching higher-order thinking skills.” Information on teachers’ responses to these two questions is presented in Tables 2.7 and 2.8 and Figures 2.6 through 2.9. The reader should keep in mind that although the responses are an indication of the quantity of teachers who have received training on these topics, they are not an indication of the quality or intensity of the training received.

In 1996, as shown in Table 2.7, 81 percent of fourth-grade students and 76 percent of eighth-grade students had teachers who reported participating in professional development on the use of technology such as computers. A higher percentage of students enrolled in algebra than students enrolled in eighth-grade mathematics had teachers who indicated that they had participated in such professional development activities sometime during the past five years.

**Table 2.7**

***Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Use of Technology, Grades 4 and 8, 1996***



		Teacher Had Professional Development in Past Five Years			
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>					
	All Students	81	225	19	221
<b>Grade 8</b>					
	All Students	76	274	24	270
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	71	264	29	260
	Pre-Algebra	76	271	24	271
	Algebra	84	297	16	294

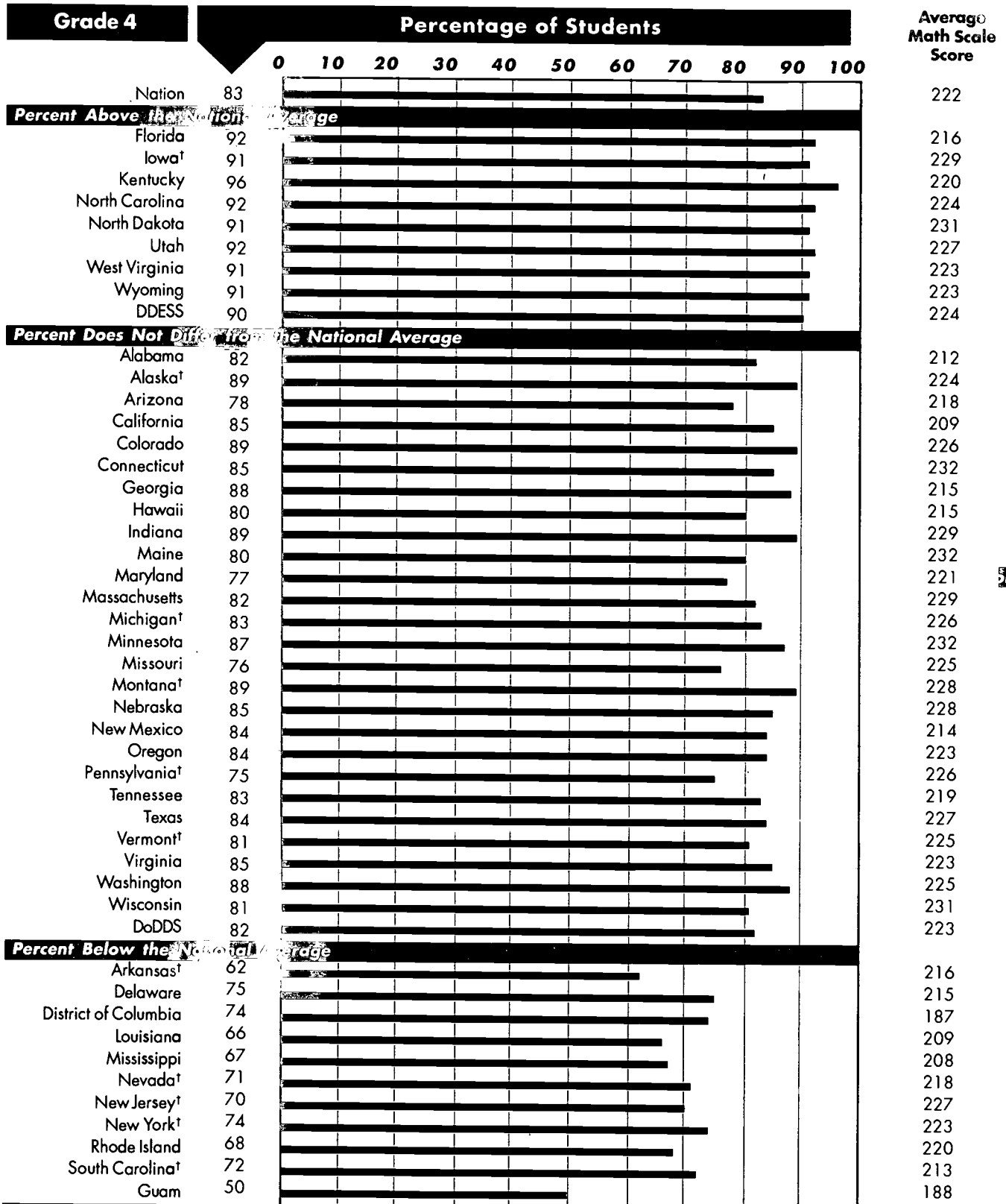
NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

Data from the NAEP 1996 state assessment on this topic are presented in Figures 2.6 and 2.7. The data on fourth-grade students show that, compared to the national average, 9 of the 47 jurisdictions had higher percentages and 11 jurisdictions had lower percentages of students whose teachers reported participating in professional development on the use of technology such as computers sometime during the past five years than the national average. At the eighth-grade level, 11 of 44 jurisdictions had higher percentages and 4 jurisdictions had lower percentages than the national average.

Figure 2.6

**Percentage of Grade 4 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and States: Public Schools Only, 1996**

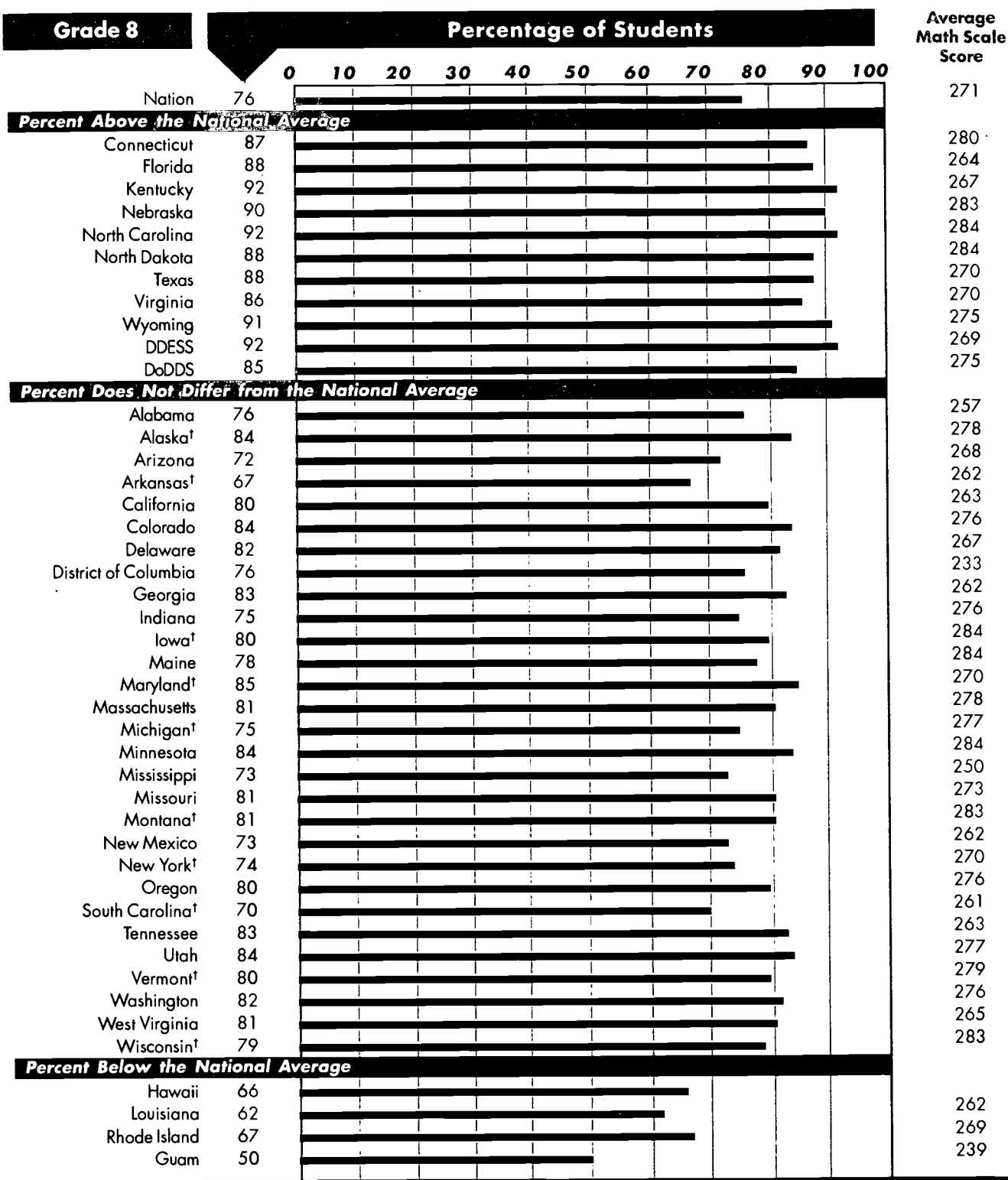


† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

Figure 2.7

**Percentage of Grade 8 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and the States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

The data in Table 2.8 show that smaller percentages of students had teachers who reported participating in professional development activities on teaching higher-order thinking skills than on use of technology such as computers. Fifty-seven percent of fourth-grade students and 47 percent of eighth-grade students were being taught mathematics by teachers who indicated that during the past five years they took courses or participated in professional development activities on teaching higher-order thinking skills. The percentage of eighth-grade students whose teachers reported participating in such professional development was not found to be related to the types of mathematics course in which student were enrolled.

**Table 2.8**

***Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, Grades 4 and 8, 1996***



		Teacher Had Professional Development in Past Five Years			
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>					
	All Students	57	225	43	222
<b>Grade 8</b>					
	All Students	47	275	53	272
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	48	265	52	262
	Pre-Algebra	48	273	52	269
	Algebra	46	299	54	295

NOTE: Information in this table is for both public and nonpublic students.

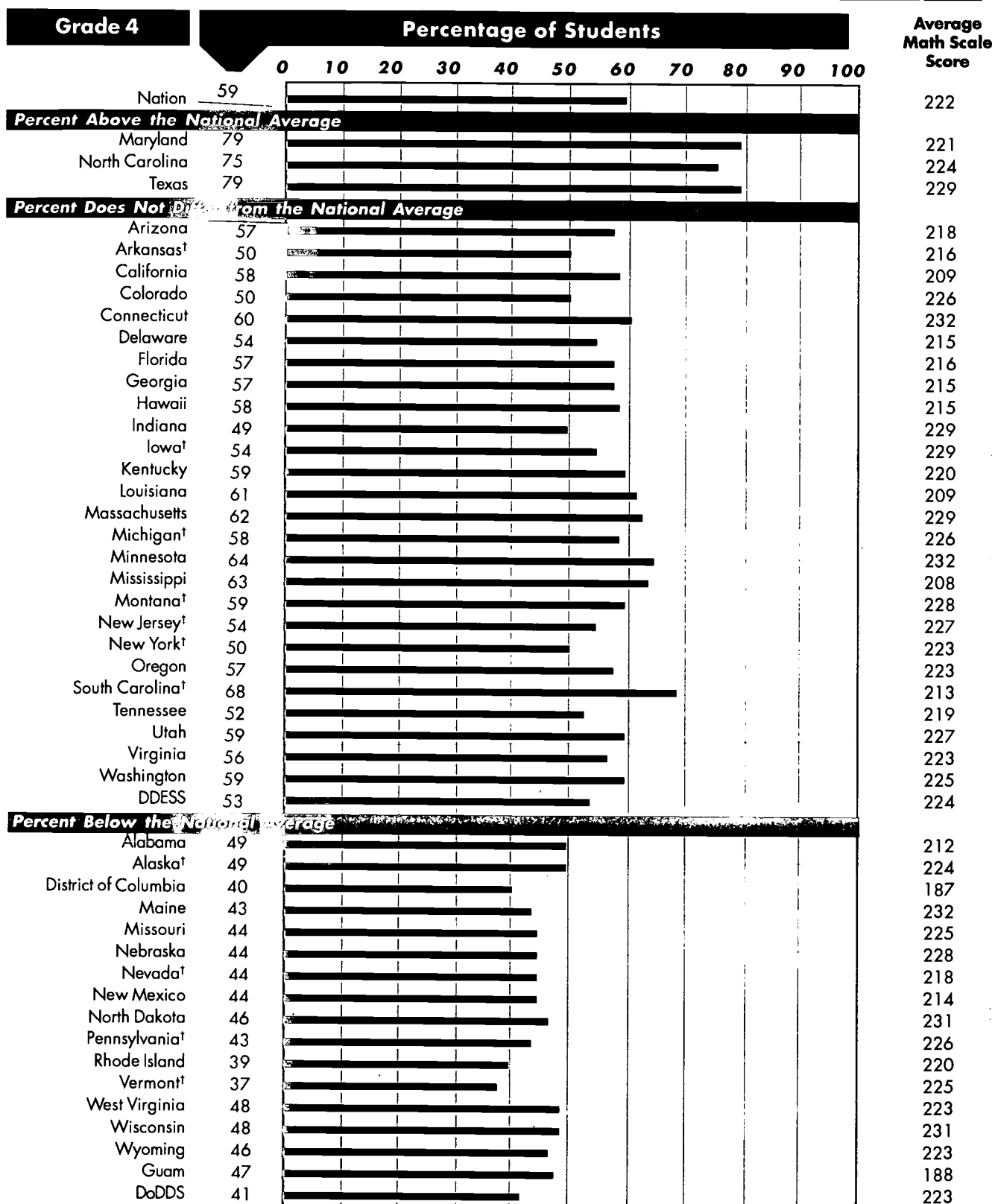
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

Figure 2.8 shows that 3 of the 47 jurisdictions had percentages of fourth-grade students whose teachers reported participating in professional development on teaching higher-order thinking skills that were higher than the national average and 17 jurisdictions had percentages of students that were lower. At the eighth-grade level, the data presented in Figure 2.9 show that 5 of the 44 jurisdictions had percentages of students higher than the national percentage while 10 of the jurisdictions had percentages of students lower than the national percentage.

**BEST COPY AVAILABLE**

Figure 2.8

# **Percentage of Grade 4 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996**



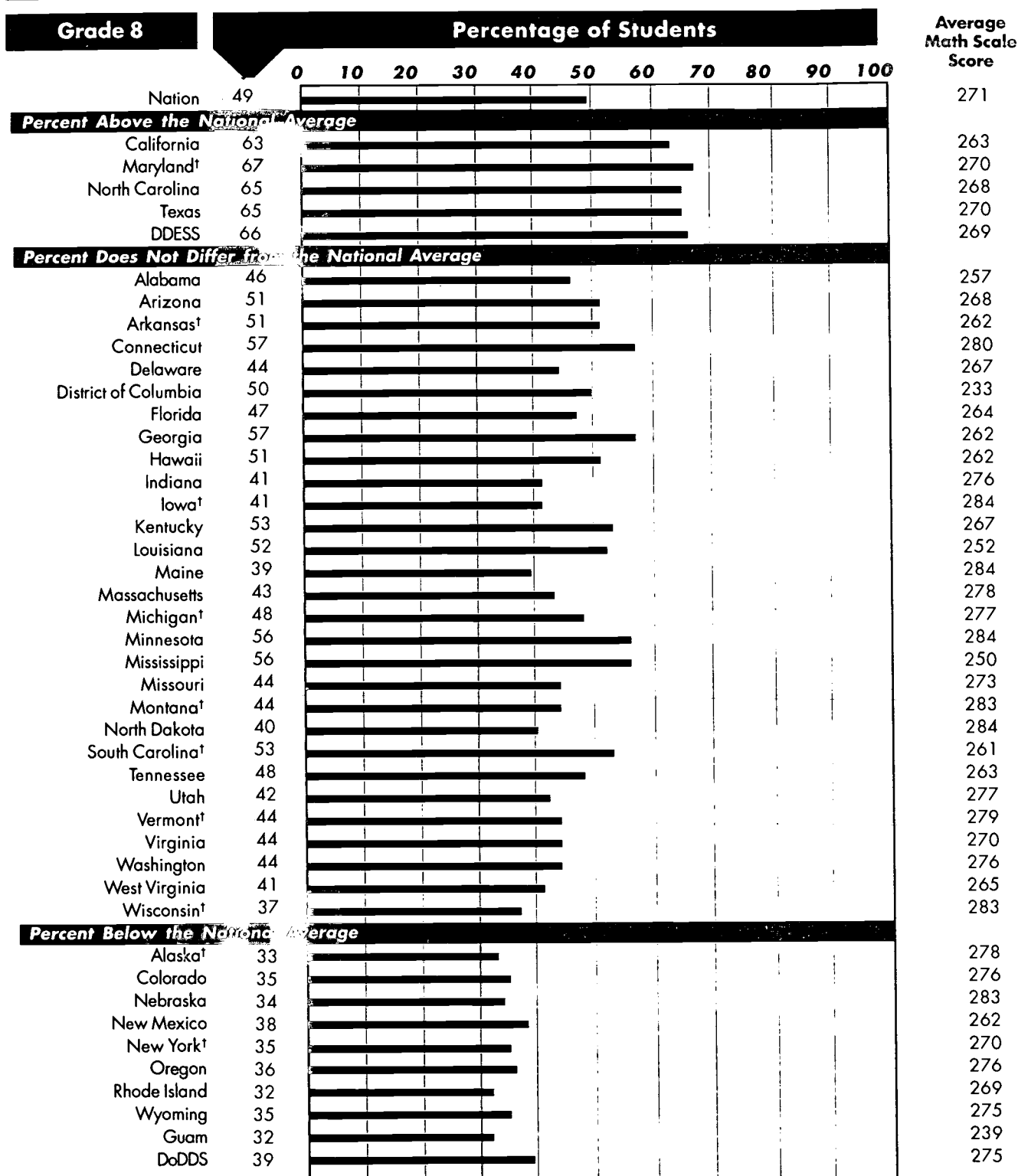
† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.



Figure 2.9

**Percentage of Grade 8 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

One of the more traditional forms of continuing professional development for teachers is attendance in courses offered by colleges and universities. In recent years, however, more professional development opportunities are being provided internally (i.e., through district and school efforts) or through collaboration with institutions of higher education in forums other than the usual college course. Therefore, it is likely that many teachers are getting some or all of their professional development through activities in their own schools. Certainly the data in Table 2.9 suggest that currently few teachers use college courses as a major means of continuing education. Teachers were asked to provide information on the number of college courses in mathematics or mathematics education they had taken during the last two years. Almost 80 percent of fourth-grade students and nearly three-fourths of eighth-grade students were taught mathematics by teachers who had taken no college courses in mathematics or mathematics education during the past two years. When results were examined by student course-taking at grade 8, similar patterns were observed within each of the course groups.

**Table 2.9**

**Percentage of Students and Average Scale Score by Number of College Courses in Mathematics or Mathematics Education Teachers Have Taken During the Last Two Years, Grades 4 and 8, 1996**



		Number of College Courses					
		Three or More Courses		One or Two Courses		No Courses	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>							
	All Students	3	215	18	224	79	225
<b>Grade 8</b>							
	All Students	11	272	15	273	74	274
<b>Students Enrolled in:</b>							
	Eighth-Grade Mathematics	10	266	14	260	76	264
	Pre-Algebra	11	269	16	274	74	271
	Algebra	12	287	16	297	72	298

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

BEST COPY AVAILABLE

# Knowledge of National Council of Teachers of Mathematics (NCTM) Standards

The process of reforming classroom instruction is facilitated by teachers' understanding and accepting that the changes they are being asked to make will improve teaching and learning.<sup>25</sup> The mathematics curriculum and evaluation standards advocated by the NCTM provided support for promoting teacher understanding of much of what constituted reform in mathematics education.<sup>26</sup> In the 1996 NAEP administration, teachers were asked about their knowledge of NCTM standards. Information on their responses is presented in Table 2.10.

**Table 2.10**

**Percentage of Students and Average Scale Score by Teachers' Knowledge of NCTM Curriculum and Evaluation Standards, Grades 4 and 8, 1996**



		Level of Knowledge							
		Very Knowledgeable		Knowledgeable		Somewhat Knowledgeable		Little or No Knowledge	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
	All Students	5	236	17	223	32	224	46	223
<b>Grade 8</b>									
	All Students	16	282	32	276	33	270	19	267
<b>Students Enrolled in:</b>									
	Eighth-Grade Mathematics	13	270	36	266	31	259	20	260
	Pre-Algebra	18	276	27	275	37	269	18	263
	Algebra	19	303	32	297	31	297	17	290

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

<sup>25</sup> Lieberman, A. (1995). Practices that support teacher development. *Teacher Learning: New Policies, New Practices*. McLaughlin, M.W. and Oberman, I. (Eds). (1996). New York: Teachers College, Columbia University; National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

<sup>26</sup> NAEP Mathematics Consensus Project. (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: National Assessment Governing Board; National Council of Teachers of Mathematics (1989). op. cit.

In 1996, 46 percent of fourth-grade students were taught mathematics by teachers who indicated having little or no knowledge of NCTM curriculum and evaluation standards, and only 5 percent had teachers who indicated being very knowledgeable. Although the average scale score of students whose teachers indicated being very knowledgeable appeared to be higher than those of other students, the apparent differences were not statistically significant.

At the eighth-grade level, 16 percent of students were taught by teachers who indicated being very knowledgeable about NCTM standards and 19 percent had teachers who indicated having little or no knowledge. Perhaps one of the reasons that eighth-grade teachers are more knowledgeable than fourth-grade teachers is simply that more eighth-grade teachers are certified mathematics teachers. The level of their teacher's knowledge on this topic appears not to have been related to the mathematics course students were taking.

Although it is not possible to determine the direction of, or even if there exists a causal relationship, eighth-grade students taught by teachers who indicated being knowledgeable or very knowledgeable had average scale scores on the 1996 NAEP mathematics assessment that were significantly higher than the average scale score of students taught by teachers who indicated having little or no knowledge. In addition, students whose teachers indicated being very knowledgeable also outperformed students whose teachers indicated being somewhat knowledgeable.

## **Summary**

In the classroom, teachers lead the learning of mathematics. This is true whether they directly impart knowledge or adopt an approach that is designed to facilitate students' construction of their own knowledge. Most teachers use a combination of these strategies.<sup>27</sup> Teachers bring a variety of resources to the task of teaching mathematics. Some of these resources have been the focus of this chapter. They include teachers' academic preparation, specifically, their undergraduate and graduate majors and exposure to college courses in a variety of mathematics topics; teaching certification; experience in teaching in general and, specifically, in teaching mathematics; professional development; and knowledge of the NCTM curriculum and evaluation standards in mathematics.

### **Grade 4 mathematics**

In 1996, mathematics or mathematics education majors were uncommon among fourth-grade teachers responsible for instruction in mathematics. Rather, four out of five fourth-grade students were being taught mathematics by teachers with undergraduate or graduate majors in education, including elementary education or secondary education; this distribution of majors had not changed significantly from 1992. Being taught by teachers with a major in mathematics, did not appear to be related to higher student performance on the 1996 NAEP mathematics assessment. However, students whose teachers reported a major in mathematics education or education performed better than students whose teachers reported a major in an "other" field.

---

<sup>27</sup> More information about classroom instructional practices is presented in a companion report on student performance on the NAEP mathematics content strands, student course-taking, and classroom practices (forthcoming).

The high percentage of students being taught mathematics by teachers with education majors is perhaps not surprising given that most fourth-grade teachers teach in self-contained classrooms and are expected to be generalists when it comes to disciplinary content. However, there are other steps that teacher trainees can take to ensure themselves of the solid subject area background that research has shown facilitates better instruction. In preparation for teaching, for example, teachers can take college courses in mathematics. The 1996 data, though, appear to show that, being generalists, teachers of fourth-grade students also tended not to take college-level mathematics courses except for courses in methods of teaching elementary mathematics. Teachers of four out of five fourth-grade students reported having taken such methods courses. Much smaller percentages of students were being taught by teachers who had taken one or more courses in the following mathematics content areas: number systems and numeration, measurement, geometry, college algebra, probability/statistics, calculus, or abstract/linear algebra. In general, the percentages of students being taught by teachers with course taking in the different mathematics content areas did not change significantly between 1992 and 1996. The exceptions were methods of teaching elementary mathematics, number systems and numeration, and abstract/linear algebra; the extent of exposure fell in these three content areas during this period.

Fourth-grade teachers of mathematics were also asked to indicate the type of teaching certificates they held. In 1996, nearly a third of fourth-grade students were being taught mathematics by teachers with a teaching certificate in mathematics, mathematics education, or elementary mathematics. Among the remaining two-thirds of students, nearly all were being taught by teachers with a teaching certificate in education but not in mathematics. The type of teaching certification their teachers held was found not to be related to students' performance on the 1996 NAEP mathematics assessment.

Like formal academic preparation and certification, years of teaching mathematics is another potential indicator of instructional quality. The relationship is less straightforward, however, as longer tenure could be associated with a broader, experience-based repertoire of instructional strategies, but it could also signal less exposure to current concepts about teaching and learning. In 1996, 48 percent of fourth-grade students were being taught mathematics by teachers with more than 10 years of experience teaching that subject area. On the 1996 NAEP mathematics assessment, students whose teachers reported 25 or more years or 6–10 years of teaching mathematics had significantly higher average scale scores than students whose teachers reported 5 or fewer years of teaching mathematics.

After they have completed their formal preparation, one of the major ways that teachers gain the knowledge and expertise to improve their teaching is through continuing professional development. In 1996, 27 percent of fourth-grade students were being taught mathematics by teachers who indicated having had more than 15 hours of professional development in mathematics or mathematics education during the past year. However, because fourth-grade teachers tend to be generalist and may, therefore, participate in professional development that cross-cut academic disciplines, this percentage may be understating the amount of professional development these teachers received. Nevertheless, student performance on the 1996 NAEP mathematics assessment was found not to be related to the hours of professional development in mathematics or mathematics education their teachers received.

In light of the prominence of the NCTM curriculum and evaluation standards in the current reform movements in mathematics, it is somewhat surprising that few fourth-grade students were taught mathematics by teachers who indicated having more than limited knowledge of NCTM curriculum and evaluation standards: only 22 percent of students had teachers who reported being knowledgeable or very knowledgeable about NCTM standards. NAEP performance of fourth-grade students was not found to be related to their teachers' depth of knowledge of NCTM standards.

### ***Grade 8 mathematics***

The teachers of the eighth-grade students were somewhat different from the teachers of the fourth-grade students. In large part this is because teachers of mathematics at grade 8 are more likely to be mathematics specialists than teachers of mathematics at grade 4. In 1996, at the eighth-grade level, 62 percent of students were being taught mathematics by teachers with an undergraduate or graduate major in mathematics or mathematics education. Student performance data from the 1996 NAEP mathematics assessment show that eighth-grade students taught mathematics by teachers with an undergraduate or graduate major in mathematics outperformed students taught by teachers whose majors were in education or some "other" field. One could surmise that teachers with a major in mathematics or mathematics education were differentially assigned to higher level mathematics courses and that, therefore, their students would necessarily outperform other students on the 1996 NAEP mathematics assessment. On the contrary, the data show no significant relationship between the teachers' undergraduate or graduate major and the type of mathematics course (algebra, pre-algebra, or eighth-grade mathematics) in which the students were enrolled.

As perhaps might be expected, given that a higher percentage of eighth-grade students had teachers with majors in mathematics, academic course work in the mathematics content areas was also more prevalent among the teachers of eighth-grade students. In 1996, more than half of the eighth-grade students were being taught by teachers who had had one or more college courses in geometry, college algebra, probability/statistics, calculus, and abstract/linear algebra. Comparisons with 1992 show significant changes only in that, in 1996, smaller percentages of students were being taught by teachers who had been exposed to one or more courses in methods of teaching elementary mathematics or in abstract/linear algebra.

While 62 percent of eighth-grade students in 1996 were being taught mathematics by teachers with undergraduate or graduate majors in mathematics or mathematics education, 81 percent were being taught mathematics by teachers with teaching certificates in mathematics. As with college majors, the types of teaching certificates held by teachers did not seem to be related to the types of eighth-grade mathematics courses they were teaching. However, eighth-grade students taught by teachers with teaching certificates in mathematics outperformed other eighth-grade students on the 1996 NAEP mathematics assessment.

In 1996, 54 percent of eighth-grade students were being taught mathematics by teachers with more than 10 years of mathematics teaching experience compared with 48 percent of fourth-grade students. Students whose teachers reported 11–24 years of experience teaching mathematics outperformed students whose teachers reported 5 or fewer years of experience.



Forty-five percent of eighth-grade students had mathematics teachers who indicated they had had more than 15 hours of professional development in mathematics or mathematics education during the past year. Teachers' hours of professional development, however, were not related to their students' performance on the 1996 NAEP mathematics assessment. In addition, teachers of different mathematics courses did not report different levels of professional development.

Finally, compared to the teachers of fourth-grade students, more mathematics teachers of eighth-grade students rated themselves as knowledgeable about NCTM curriculum and evaluation standards. In 1996, 48 percent of eighth-grade students had teachers who reported being knowledgeable or very knowledgeable about NCTM standards. Unlike at grade 4, eighth-grade students whose teachers reported being knowledgeable or very knowledgeable did better on the 1996 NAEP mathematics assessment than students whose teachers reported little or no knowledge of the standards. Additionally, students whose teachers report being very knowledgeable outperformed students whose teachers reported being somewhat knowledgeable.



## Chapter 3

# ***What Emphasis Does Mathematics Instruction Receive?***

State, district, and school policies and practices can all affect students' experiences in mathematics education. Mathematics reform efforts since the early 1980s have advocated policy changes that people believe can help improve the teaching and learning of mathematics. This chapter focuses on several policies and practices in mathematics education that can reflect high expectations and provide students with opportunities to learn. These include the development of curriculum frameworks based on high curriculum and performance standards; high school graduation requirements; mathematics course offerings; and time spent on mathematics instruction.

### ***Curriculum Frameworks***

Current reform efforts emphasize the importance of having high expectations for teaching and learning. High expectations are reflected in school policies as well as in classroom practices, and curriculum frameworks are often the vehicles used to communicate high standards and expectations. According to a report from CCSSO, in 1996, 39 of 51 jurisdictions (which include the 50 states and the District of Columbia) indicated that they had mathematics content standards ready for implementation.<sup>1</sup> In 1996, NAEP data show that principals of 80 percent of twelfth-grade students, 82 percent of eighth-grade students, and 89 percent of fourth-grade students indicated that their schools are expected to follow a district or state curriculum in mathematics.<sup>2</sup> The hope is that these district or state curricula are aligned to their district or state content standards, and that the content standards reflect high expectations.<sup>3</sup>

---

<sup>1</sup> Council of Chief State School Officers (1996). op. cit.

<sup>2</sup> The source of these data is the 1996 NAEP mathematics assessment.

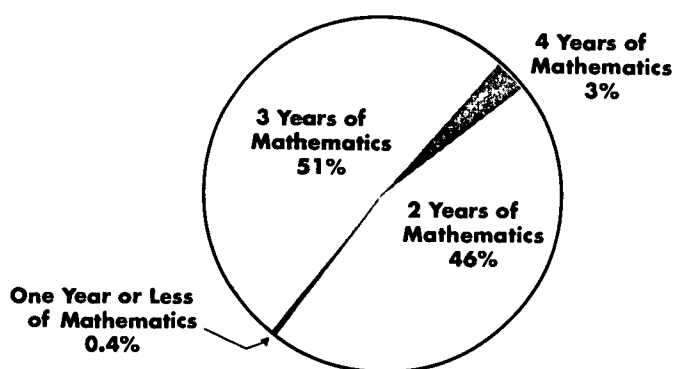
<sup>3</sup> Smith, M.S., & O'Day, J. (1990). *Politics of education association yearbook 1990*, pp. 233-267. Taylor & Francis, Ltd.

## Graduation Requirements

Other school policies that can reflect high expectations include graduation requirements, such as the number of mathematics courses or Carnegie units in mathematics that students are required to complete successfully for high school graduation. School reform efforts, beginning over a decade ago with the publication of *A Nation At Risk*, have recommended increases in mathematics credits or courses required for high school graduation.<sup>4</sup> Current efforts have gone beyond number of courses to also focusing on raising the difficulty level of courses that students take.<sup>5</sup> The data presented in Figure 3.1 show that, in 1996, just over half (54%) of the nation's twelfth-grade students attended schools where at least three years of mathematics (from grades 9 through 12) were required for graduation; 46 percent of students attended schools where two years of mathematics were required; and less than one percent of students attended schools where only one year or less of mathematics was required.

**Figure 3.1**

### **Percentage of Twelfth-Grade Students by Mathematics Graduation Requirement (Grades 9 through 12), 1996**



NOTE: Information in this table is for both public and nonpublic students. Percentages do not sum to 100 percent due to rounding.  
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

In a 1996 survey of key state education policies, CCSSO found that states varied in their course credit requirements for high school graduation.<sup>6</sup> In two states, students needed a combined mathematics-and-science total of five course credits, while in 18 states, this course credit requirement was three or four credits. In only one state was the requirement as low as one course credit; in the remaining 24 states the requirement was two course credits. Reasons for the variation range from different philosophies regarding the amount of mathematics a high school

<sup>4</sup> National Commission on Excellence in Education (1983). *op. cit.*


<sup>5</sup> Chaney, B., Burgdorf, K., & Atash, N. (1997). Influencing achievement through high school graduation requirements. *Educational Evaluation and Policy Analysis*, 19(3), pp. 229-244.

<sup>6</sup> CCSSO (1996). *op. cit.*

graduate should have to the fact that, in different states, different entities set the requirements. For example, CCSSO found that in six of the states, local boards set their own graduation requirements with no state-set requirements.

With increases in the number of mathematics courses required, the hope of many states is that more mathematics will also lead to higher levels of mathematics content. To test this hypothesized relationship, data from the 1996 NAEP mathematics assessment were used to examine progression through an algebra-calculus sequence in relation to high school graduation requirements.<sup>7</sup> The hierarchical sequence of courses in the algebra-calculus sequence is supported by a study conducted by Chaney, Burgdorf, and Atash.<sup>8</sup> In the following tables, data for geometry course taking are displayed separately because of the difference in geometry content relative to algebra-calculus content.

The data in Table 3.1 show the percentage distribution of students by highest level of mathematics course taken in the algebra-calculus sequence, subset by whether their schools required 3 or 4 years, or 2 or fewer years of mathematics for high school graduation.

Table 3.1		<b>Percentage of Twelfth-Grade Students and Average Scale Score by Highest Algebra-Calculus Course Taken and Mathematics Graduation Requirement (Grades 9 through 12), 1996</b>		THE NATION'S REPORT CARD		
		<b>Mathematics Graduation Requirement:</b>				
		3 or 4 Years		2 Years or Less		
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	
<b>Highest Algebra-Calculus Course Taken:</b>						
Not Taken Pre-Algebra		21	274	5	267	
Pre-Algebra		4	273	5	280	
First-Year Algebra		20	286	25	291	
Second-Year Algebra		50	304	47	310	
Pre-Calculus/ Third-Year Algebra		16	321	12	325	
Calculus		8	342	6	340	

1 Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

<sup>7</sup> See Appendix A for an explanation of the algebra-calculus sequence used in this report.

<sup>8</sup> Chaney, Burgdorf, & Atash (1997). op. cit.

Table 3.2 shows the percentage distribution of students who have taken a geometry course, also by whether their schools required 3 or 4 years, or 2 or fewer years of mathematics. For purposes of this analysis, students were credited with having taken a course only if they reported that they had taken a course of at least one school year's duration.

**Table 3.2** *Percentage of Twelfth-Grade Students Who Have Taken Geometry by Mathematics Graduation Requirement (Grades 9 through 12), 1996*



Mathematics Graduation Requirement			
3 or 4 Years		2 Years or Less	
Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
86	309	76	312

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

There appears to be little relationship between the percentages of twelfth-grade students at the different levels of courses in the algebra-calculus sequence and the schools' mathematics graduation requirement. That is, regardless of the graduation requirement, the highest percentage of students indicated that second-year algebra was the highest course they had taken. Chaney, Burgdorf, and Atash found that graduation requirements appeared to affect student course-taking only for some students.<sup>9</sup> The authors found that most students (64 percent for mathematics) took more courses than were required for graduation and speculated that these students were motivated to take courses by factors other than graduation requirements. In addition, the policy of increasing graduation requirements may be perceived as necessary in districts where students may tend to opt out of mathematics courses if not required while not perceived as necessary in districts where high percentages of students go to colleges or universities that require more mathematics for favorable admissions.

Data from the 1996 NAEP mathematics assessment show that although only three percent of twelfth-grade students attended schools that required four years of mathematics for graduation, 48 percent indicated having taking seven or more semesters of mathematics.<sup>10</sup> As perhaps expected, with few exceptions, regardless of whether students attended schools that required 3 or 4 years or 2 or fewer years of mathematics for graduation, students enrolled in more advanced mathematics classes performed better on the 1996 NAEP mathematics assessment than students in less advanced classes. Furthermore, only in the second-year algebra category was the difference in performance by mathematics graduation requirement statistically significant:

<sup>9</sup> Ibid.

<sup>10</sup> NAEP, 1996 mathematics assessment.

students in schools that required two or fewer years outperformed students in schools that required 3 or 4 years of mathematics. The reason for this significant difference is not clear particularly because the percentage of students whose highest course was second-year algebra in schools that required two or fewer years was not significantly different from those who attended schools that required 3 or 4 years of mathematics. Some students who took second-year algebra in schools that required two or fewer years of mathematics may have been highly motivated to perform well in this advanced course and, subsequently, on the NAEP assessment.

An examination of student responses regarding geometry courses shows that regardless of whether schools required three or four years, or two or fewer years of mathematics, a large majority of students indicated having taken one or more years of geometry. Chaney, Burgdorf, and Atash found that mathematics courses were highly sequential and that students generally took geometry following first-year algebra.<sup>11</sup> Therefore, if students began the ninth grade taking first-year algebra, they would typically have taken geometry in their second year. In any school where the mathematics requirement was two or more years, most students would reasonably have taken a year or more of geometry. The percentage of students who took geometry in schools that required three or four years of mathematics was significantly higher than the percentage of students who took geometry at schools that required two or fewer years of mathematics for graduation.

## **Course Offerings**

The nation's students are being expected to learn more mathematics, and higher levels of mathematics, during their years of elementary and secondary education. In order for students to meet these two goals, the schools they attend must offer them opportunities to access more and higher level mathematics courses. To a great extent, schools appear to be doing so. For example, NAEP data indicate that in 1996, only one percent of twelfth-grade students were attending schools that provided no advanced mathematics courses of at least one semester in length.<sup>12</sup> Table 3.3 presents information about the frequency of specific higher level mathematics courses that were taught in the nation's schools in 1996 and 1992.

In 1996, over 80 percent of twelfth-grade students attended schools that offered at least one semester of the following advanced mathematics courses: trigonometry; pre-calculus, third-year algebra, elementary functions, or analysis; and calculus. About a third of the nation's twelfth-grade students were attending schools that offered probability and/or statistics.<sup>13</sup> For all of the advanced mathematics courses listed in Table 3.3, the percentages of students in schools offering these advanced courses did not change significantly from 1992 to 1996.

---

<sup>11</sup> Chaney, Burgdorf, & Atash. (1997). *op. cit.*

<sup>12</sup> The source of these data is the 1996 NAEP mathematics assessment.

<sup>13</sup> Some schools also offered a unified, integrated, or sequential mathematics program, a type of program that is typically expected to progress from basic to more advanced levels of mathematics. However, the information from NAEP does not allow us to determine the extent to which students enrolled in these unified, integrated, or sequential mathematics courses were actually being taught at an advanced level. Therefore, we have not reported these data here.

Table 3.3

**Percentage of Twelfth-Grade Students and Average Scale Score by Whether Specific Advanced Mathematics Course of One Semester in Length Taught in Their School**



	Offered		Not Offered	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Trigonometry</b>				
1996	80	304†	20	306
1992	80	300	20	297
<b>Pre-Calculus, Third-Year Algebra, Elementary Functions, Analysis</b>				
1996	84	305†	16	300
1992	88	300	12	296
<b>Calculus</b>				
1996	82	305†	18	300
1992	79	301	21	294
<b>Probability and/or Statistics</b>				
1996	32	309	68	302†
1992	29	304	71	297

† Significantly different from 1992.

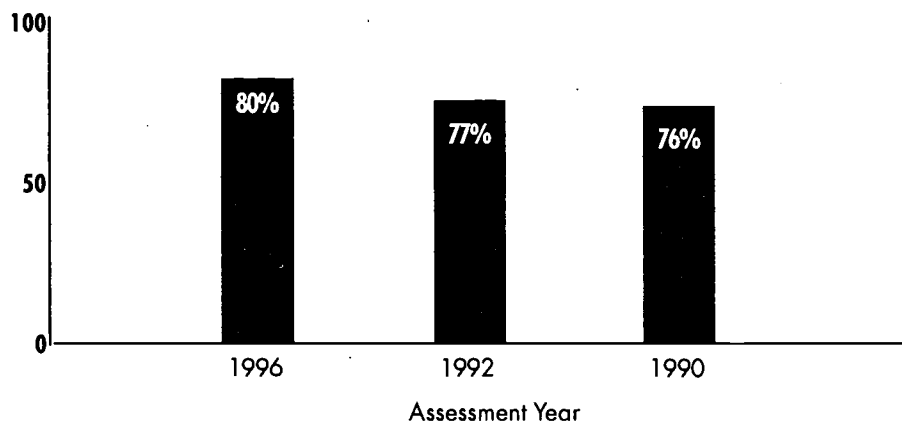
NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

Programs such as Equity 2000 and middle-school level reform initiatives have suggested that eighth-grade students should be ready for, and have the opportunity to take, first-year algebra.<sup>14</sup> Part of the rationale for these recommendations is a belief that early access to algebra will positively impact experiences and achievement in secondary mathematics.<sup>15</sup> As shown in Figure 3.2, in 1996, 80 percent of eighth-grade students attended schools that offered eighth-grade algebra for high school course placement or credit. The percentage of students in schools that offered this type of algebra course to eighth-grade students did not increase significantly from 1992 or 1990.

<sup>14</sup> Jones, V. (1994). *Lessons from the Equity 2000 education reform model*. New York: The College Board.

<sup>15</sup> Smith, J.B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis*, 18(2), 141-153.

**Figure 3.2****Percentage of Eighth-Grade Students Whose Schools Offer Algebra for High School Credit or Placement**

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996, 1992, and 1990 Mathematics Assessments.

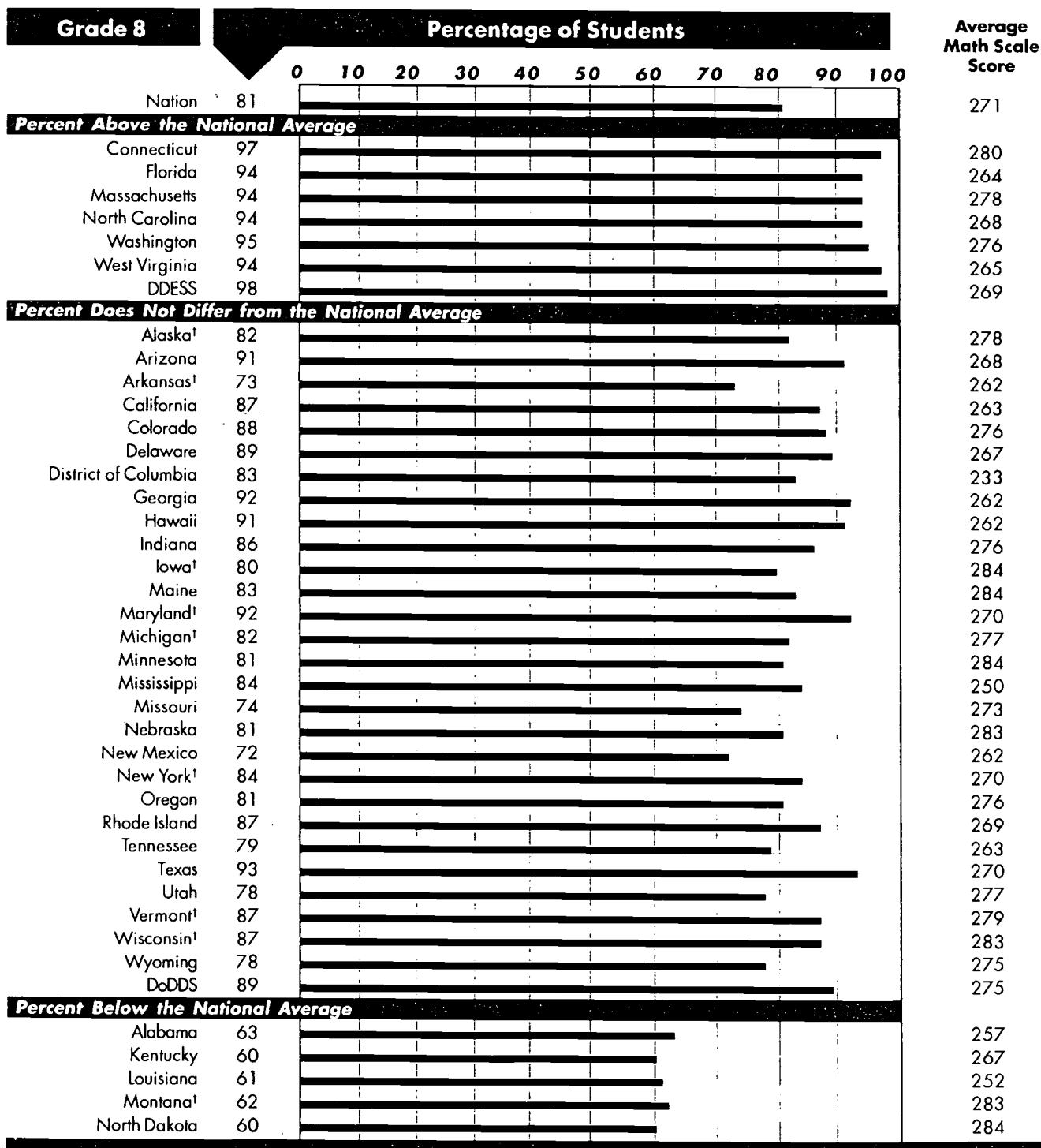
Figure 3.3 shows the variation by state in the percentage of eighth-grade students who attended schools that offered them eighth-grade algebra for high school placement or credit. In 1996, 81 percent of all U.S. public school eighth-grade students were in schools that offered such courses.<sup>16</sup> Of the jurisdictions participating in the 1996 NAEP state mathematics assessment, 7 had percentages of students higher than the national average, 29 had percentages similar to this national percentage, and 5 had percentages lower than the national percentage.

<sup>16</sup> The difference in the nation's percentage in Figures 3.2 and 3.3 is due to the fact that the percentage, 80 percent, in Figure 3.2 is based on public and nonpublic students and the percentage, 81 percent, in Figure 3.3 is based on public school students only.



Figure 3.3

**Percentage of Students in Schools That Offer Algebra for Eighth-Grade Students, for the Nation and States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

NOTE: Standard error estimates could not be accurately determined for South Carolina, Virginia, and Guam. Therefore, those jurisdictions are not listed.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

Of course, not all eighth-grade students whose schools offer algebra enroll in algebra courses. Table 3.4 presents data on the percentages of students who reported being enrolled in different eighth-grade mathematics courses. Results are presented separately for students attending schools that offered eighth-grade students high school-credit algebra and those attending schools that did not.

**Table 3.4**

***Percentage of Eighth-Grade Students by Mathematics Course Enrollment and Availability of Algebra, 1996***



Mathematics Course Enrolled in:	Algebra Offered for High School Credit/Placement			
	Yes		No	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Eighth-Grade Mathematics	39	261	56	266
Pre-Algebra	28	271	24	268
Algebra	28	298	16	279
Other Mathematics	5	276	5	***

\*\*\* Sample size insufficient to permit a reliable estimate.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996

Mathematics Assessments.

Regardless of whether or not students were enrolled in schools that provided high school-credit algebra for eighth-grade students, the largest percentage of students were enrolled in eighth-grade mathematics. However, the percentage enrolled in eighth-grade mathematics was much higher for schools that did not offer this type of algebra (56%) than at schools that did (39%). Also, perhaps as expected, the percentage of students in algebra was higher for schools that offered high school-credit algebra (28%) than for schools that did not (16%).<sup>17</sup> Nevertheless, in schools that offer algebra for high school credit, algebra students outperformed all other students and pre-algebra students outperformed students in eighth-grade mathematics, while in schools that did not offer algebra, performance on the NAEP was not related to type of course in which students were enrolled. Furthermore, only for algebra students was the performance of those who attended schools that offer algebra significantly different from those who attended schools that did not: those in schools that offered algebra performed better.

<sup>17</sup> There is no way to determine from the NAEP data whether students who indicated algebra enrollment at the latter schools were reporting in error, were actually enrolled in non-high school credit algebra courses (or courses where high school credit or placement was not automatic), or were enrolled in algebra courses offered off-campus at local high schools or community colleges.

Figure 3.4 shows that in 1996, 25 percent of the nation's eighth-grade students were enrolled in algebra. The percentage of students enrolled in algebra in 8 of the 44 jurisdictions that participated in the state assessment was significantly higher than the national percentage and the percentage of students in 5 of the 44 jurisdictions was lower than the national percentage.

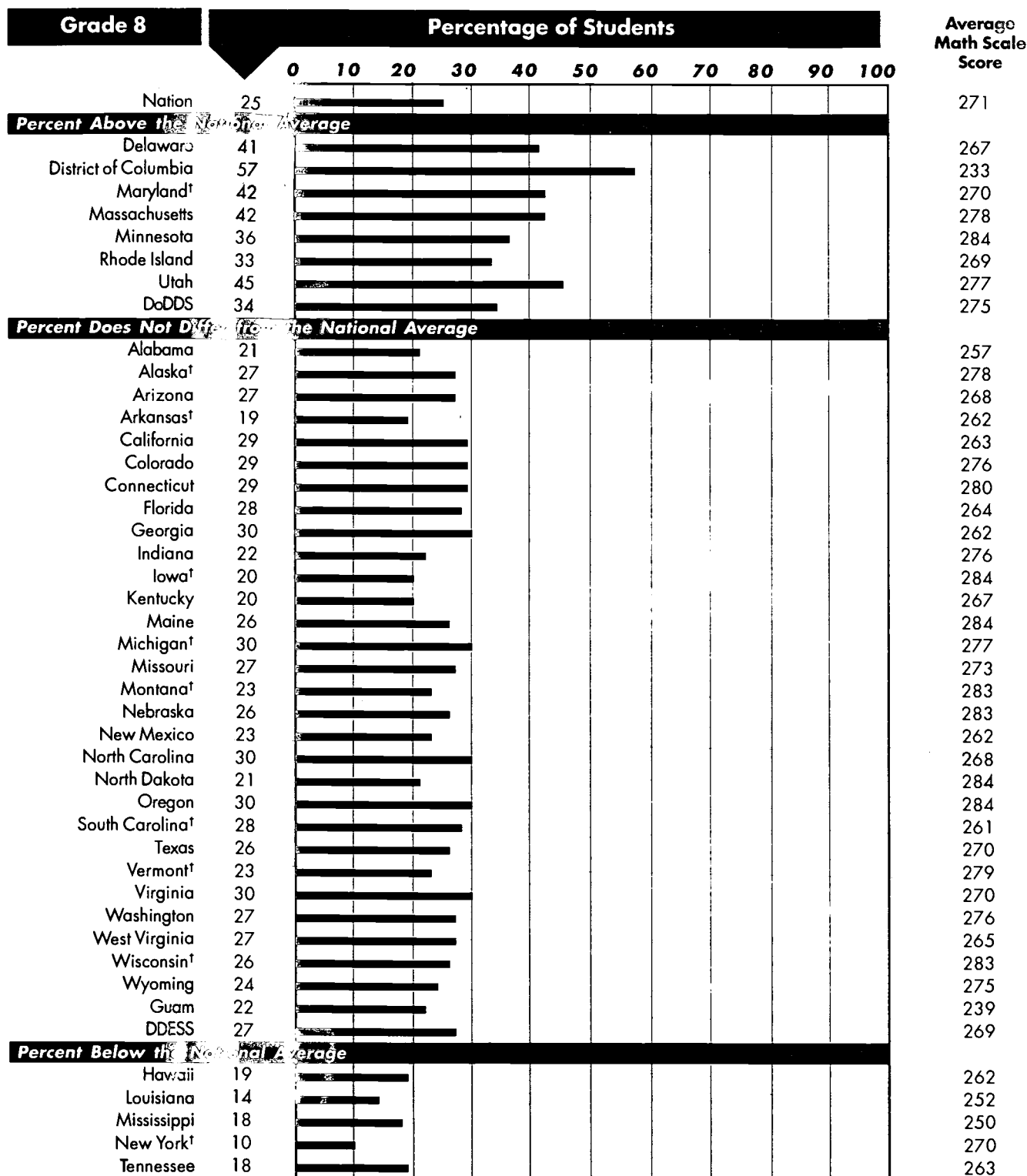
## ***Instructional Time***

The amount of time that teachers spend on mathematics instruction is another indicator of students' opportunities to learn mathematics. Although the reasons for more or less time being devoted to mathematics are varied, in general the assumption is that if more time is spent on mathematics instruction, teachers can impart more knowledge and may implement more innovative pedagogical practices such as the use of project-based instruction, or a more investigative approach to learning. The data cannot confirm, however, that teachers are necessarily taking advantage of having more instructional time or that they have the flexibility of arranging instructional minutes to best fit their needs.

Data from the NAEP survey show that 87 percent of the nation's eighth-grade students in 1996 attended schools that were departmentalized for instruction. Teachers in these types of schools probably have little, if any, discretion on the amount of time they spend on instruction. At the fourth-grade level, however, although teachers generally must also adhere to policies set by state or local boards of education regarding numbers of minutes of instruction, they have more discretion to increase the amount of weekly time spent on mathematics by using strategies such as integrated-curricular instruction.

Figure 3.4

# Percentage of Students Enrolled in Algebra, for the Nation and the States: Public Schools Only, 1996



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

Information about instructional time is presented in Table 3.5. In 1996, more than two-thirds (68%) of fourth-grade students were in classes in which teachers indicated spending four or more hours per week on mathematics instruction.<sup>18</sup> Only a small percentage (6%) had teachers who reported spending 2.5 hours or less per week. The amount of time spent weekly on mathematics instruction did not appear to be related to student performance on the 1996 NAEP mathematics assessment.

**Table 3.5**

**Percentage of Students and Average Scale Score by Time on Mathematics Instruction, Grades 4 and 8**



		Time Spent Weekly on Mathematics					
		Two and One-Half Hours or Less		More than Two and One-Half Hours, But Less than Four Hours		Four Hours or More	
	Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>							
All Students	1996	6	228	26	226	68	223
	1992	5	224	25	224	71	217
<b>Grade 8</b>							
All Students	1996	20	269	47	275	33	274
	1992	13	270	55	270	32	268
<b>Students Enrolled in:</b>							
Eighth-Grade Mathematics	1996	25†	263	49	266	26	260
	1992	12	257	58	258	31	255
Pre-Algebra	1996	16	268	49	275	35	266
	1992	15	273	53	274	32	270
Algebra	1996	15	293	41	298	44	298
	1992	11	304	55	300	34	299

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

<sup>18</sup> NAEP survey categories do not distinguish amount of time spent beyond four hours per week.

The percentages of fourth-grade students exposed to the different levels of hours of mathematics instruction did not change from 1992 to 1996. For example, in 1996, 68 percent of students were taught by teachers who indicated spending four or more hours a week on mathematics instruction, while in 1992 the percentage was 71 percent; this difference was not statistically significant.

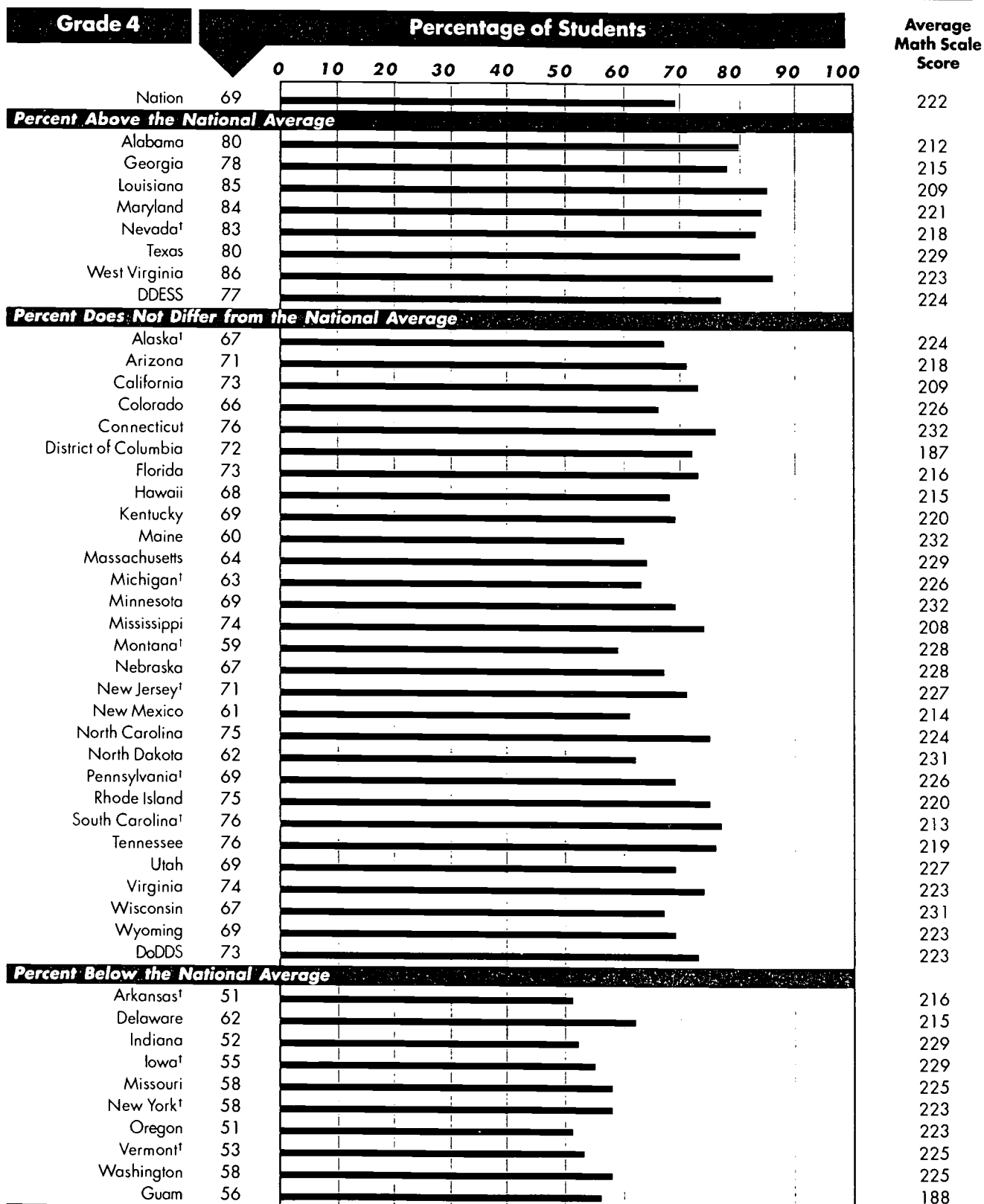
In 1996, when no distinctions are made regarding variations in time above four hours a week, the percentage of time spent on mathematics instruction at grade 8 appears to be somewhat more varied than at grade 4. Almost half of eighth-grade students (47%) were taught by teachers who indicated spending more than 2.5, but less than 4 hours per week on mathematics instruction; 33 percent were taught by teachers who indicated spending 4 or more hours, and 20 percent were taught by teachers who indicated spending 2.5 hours or less per week. Although the percentages of students in classes where four or more hours per week were spent on mathematics instruction appears to be higher for algebra than for pre-algebra or eighth-grade mathematics, the percentages were not significantly different from each other. Over time, there were neither significant increases nor decreases in the percentage of eighth-grade students receiving a given level of hours of mathematics instruction. This was true regardless of whether students were taking pre-algebra or algebra. Somewhat puzzling, however, the percentage of students taking eighth-grade mathematics in classes where instruction was 2.5 or less hours per week was significantly higher in 1996 than it was in 1992.

Figures 3.5 and 3.6 show state data on the percentages of fourth- and eighth-grade students who received four or more hours of mathematics instruction per week. At both grade levels, the majority of jurisdictions had percentages of students receiving four or more hours of mathematics instruction per week that were similar to the national percentage for that grade level.

BEST COPY AVAILABLE

Figure 3.5

**Percentage of Grade 4 Students with Four or More Hours per Week of Mathematics Instruction, for the Nation and States: Public Schools Only, 1996**



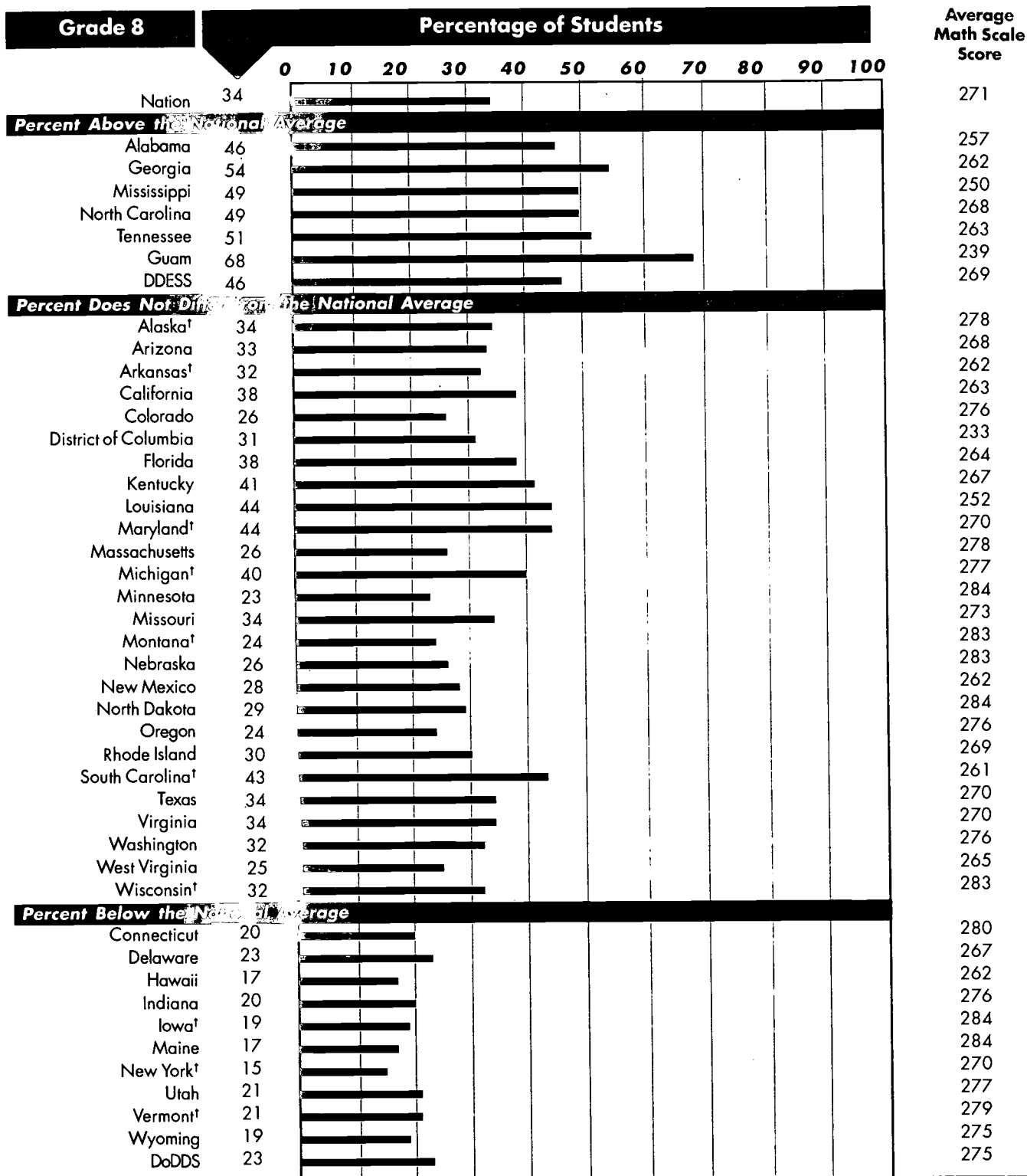
† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.



Figure 3.6

**Percentage of Grade 8 Students with Four or More Hours per Week of Mathematics Instruction, for the Nation and States: Public Schools Only, 1996**



† Jurisdiction did not satisfy one or more of the 1996 school participation rate guidelines for the school sample(s) presented in this table (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics State Assessment.

## **Summary**

Students' experiences in mathematics education can be influenced by their opportunities to learn. These opportunities in turn are influenced by school policies and practices. In this chapter we reported on a number of school policies and practices that can influence mathematics education: in particular, curriculum frameworks, mathematics graduation requirements, mathematics courses offered to students, and amount of time spent weekly on mathematics instruction.

One of the major components of current education reform is the development and implementation of curriculum frameworks that reflect high academic standards and guide classroom instruction and assessment. In 1996, at least 80 percent of our nation's students, in each of grades 4, 8, and 12, attended schools in which teachers were expected to follow a district or state curriculum in mathematics.

In 1996, just over half of twelfth-grade students attended schools that required at least three years of mathematics courses for high school graduation. Nearly all of the remainder (46 percent) attended schools that required two years of mathematics for graduation. Simply requiring more mathematics courses, however, does not insure that students will enroll in higher level courses with more challenging content. In fact, in 1996, regardless of whether students attended schools that required 3 or 4 years or 2 or fewer years of mathematics for high school graduation, the modal response from students as to their highest algebra-calculus course taken was second-year algebra. However, a higher percentage of students reported having taken geometry in schools that required 3 or 4 years rather than 2 or fewer years of mathematics for graduation.

Schools also need to offer students opportunities to take higher level courses. In 1996, all but one percent of twelfth-grade students attended schools that indicated they offered some advanced courses in mathematics for their students. Eighty percent or more of twelfth-grade students were in schools that offered trigonometry, pre-calculus (or equivalent level courses such as third-year algebra, elementary functions, analysis), and/or calculus. However, about a third of students attended schools that offered courses in probability and statistics. These percentages had not changed significantly from 1992.

At the middle-school level, students are also being encouraged to take higher level mathematics, specifically algebra, as soon as they are prepared to take it. In 1996, 80 percent of eighth-grade students attended schools that offered algebra to eighth-grade students for high school credit or placement. This percentage had not changed significantly from 1992 or 1990.

There appears to be some significant differences in course-taking patterns and performance on the 1996 NAEP mathematics assessment between students who attended schools that offered eighth-grade algebra for high school credit or placement and students who attended schools that did not. For example, the percentage of students enrolled in eighth-grade mathematics was higher for schools that did not offer algebra (56%) than for schools that did

(39%), although in both types of schools, the modal response from students was enrollment in eighth-grade mathematics.

In addition to the types of courses they offer, schools can, in theory, encourage more mathematics learning by increasing the amount of time spent on mathematics instruction. In 1996, 47 percent of eighth-grade students were in classes in which mathematics was taught more than 2.5, but less than 4, hours per week. Another 33 percent of students received 4 or more hours of instruction each week, while 20 percent were in classes with 2.5 hours or less of weekly instruction. These 1996 percentages were not significantly different from 1992 percentages.

At the fourth-grade level, 1996 NAEP data show 68 percent of students receiving four or more hours of weekly mathematics instruction. Twenty-six percent received more than 2.5, but less than 4, hours of weekly instruction, and only six percent received 2.5 hours or less. The amount of time spent weekly on mathematics instruction was not related to students' mathematics performance. The percentages of students in each of the instructional-hour categories did not change significantly from 1992 to 1996.

## Chapter 4

# ***What Are the Resources in Schools That Support Mathematics Learning?***

School resources take different forms. Some are specific to mathematics, but many are general resources that improve learning opportunities across the curriculum. They all, however, can be used together to support mathematics teaching and learning.<sup>1</sup>

### ***Availability of Resources***

One way in which schools support teaching and learning is by providing teachers and their classrooms with the instructional materials and other resources that they need for delivering and improving mathematics instruction. Table 4.1 presents data on teachers' perceptions of the extent to which their schools provide needed resources.

In 1996, 53 percent of fourth-grade students were taught mathematics by teachers who indicated that they got most of the instructional materials and other resources they needed to teach their class, 34 percent of students were taught by teachers who indicated that they received some or none of the resources needed, and 13 percent were in classes where teachers indicated that they received all of the resources needed. Teachers' perceptions of the adequacy of resources available in their classes were not associated with student performance on the 1996 NAEP mathematics assessment. Further, the percentages of fourth-grade students in classes with differing levels of access to needed resources did not change in 1996 compared to 1992 or 1990. Regardless of level of resources, however, students in 1996 outperformed students in 1990. While adequate resources may be important in providing effective mathematics instruction, there are many other factors influencing student achievement in mathematics.

---

<sup>1</sup> Hanushek, E. A. (1997). op cit.; Spillane, J.P. & Thompson, C.L. (1997). op. cit.

Table 4.1

**Percentage of Students and Average Scale Score by Teachers' Reports on the Availability of Resources, Grades 4 and 8**



		Availability of Resources					
		I Get All the Resources I Need		I Get Most of the Resources I Need		I Get Some or None of the Resources I Need	
Assessment Year		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4*</b>							
All Students	1996	13	228*	53	225*	34	222*
	1992	11	223	52	223*	37	216*
	1990	14	216	49	217	37	209
<b>Grade 8</b>							
All Students	1996	21	281*	58	273	21†	266
	1992	14	274	53	271	33	264
	1990	15	265	54	266	31	260
<b>Students Enrolled in:</b>							
Eighth-Grade Mathematics	1996	22	269*	57	263†	22†	258
	1992	14	261	51	256	35	256*
	1990	15	258	54	255	31	250
Pre-Algebra	1996	19	277	58	271	23	265
	1992	13	277	53	275	34	268
	1990	11	275	54	274	34	272
Algebra	1996	23	305	57	297	20	288
	1992	15	305	59	302	26	290
	1990	17	290	56	299	28	292

\* Significantly different from 1990.

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996, 1992, and 1990 Mathematics Assessments.

Fifty-eight percent of eighth-grade students in 1996 had teachers who reported receiving most of the instructional materials and other resources they need, 21 percent of students were in classes that received some or none of the resources teachers felt were needed, and another 21 percent of students were in classes that received all of the resources teachers felt were needed. Unlike the results at the fourth-grade level, eighth-grade students in classes that got all of the resources needed performed better on the 1996 NAEP mathematics assessment than did students in classes that got most or some or none of the resources needed and students in classes that got most of the resources needed outperformed students in classes that got some or none of the resources needed.

In each of the different types of eighth-grade mathematics classes, the majority of students were in classes that got most of the resources they needed. Only in algebra classes, however, was the level of access to resources associated with student performance: students in classes that received all of the resources needed outperformed students in classes that received only some or none of the resources needed.

In 1990, 1992, and 1996 similar percentages of eighth-grade students were in classes that received all or most of the resources needed. However, a lower percentage of students were in classes that received some or none of the resources they needed in 1996 (21%) than in 1992 (33%). Initiatives to improve mathematics and science education such as the National Science Foundation's (NSF) Statewide Systemic Initiative (SSI) and Urban Systemic Initiative (USI) programs may have positively influenced the availability of resources needed to teach mathematics using currently recommended instructional strategies.<sup>2</sup>

---


<sup>2</sup> Information on NSF's SSI and USI programs can be obtained from the website <<http://www.ehr.nsf.gov/EHR/ESR/index.htm>>.

## Curriculum Specialists and Planning Time

As schools and teachers begin to implement instructional strategies advocated by current educational reform movements in mathematics, teachers find themselves needing more time for lesson preparation and also help in planning and implementing new and more effective lessons.<sup>3</sup>

*Curriculum specialists* can be a significant resource in instructional planning and curriculum plan implementation. Information on teachers' perceived access to curriculum specialists for help and assistance in mathematics is provided in Table 4.2. In 1996, 43 percent of fourth-grade students were taught by teachers who reported having access to curriculum specialists in mathematics. Comparisons between 1992 and 1996 show that there were no significant changes in the percentages of students taught by teachers with access to curriculum specialists.

In 1996, almost half of eighth-grade students were taught by teachers who indicated that curriculum specialists in mathematics were available to them. No comparable data were available for 1992. Grades that are departmentalized, as most schools in the assessment reported that their eighth-grades were, typically have a department chairperson, who in some schools serves the role of a curriculum specialist. Consequently, the reader must consider that a positive response to this question could mean the availability of different types of curriculum specialists to teachers in schools at different grade levels.

Table 4.2: <b>Percentage of Students and Average Scale Score by Whether Teachers Have Access to Mathematics Curriculum Specialists, Grades 4 and 8</b>		THE NATION'S REPORT CARD 			
		Have Access		Do Not Have Access:	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4†</b>					
	1996	43	222	57	226†
	1992	47	219	53	221
<b>Grade 8†</b>					
	1996	49	270	51	277

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

<sup>3</sup> Darling-Hammond, L. & McLaughlin, M.W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan*, 76(8), pp. 597-604; Miles, K.H. (1995). Freeing resources for improving schools: A case study of teacher allocation in Boston Public Schools. *Educational Evaluation and Policy Analysis*, 17(4), pp. 476-493.



Furthermore, the reader should bear in mind that curriculum specialists are resources that not all schools are able to provide on-site at all times. In some districts, curriculum specialists reside at the central office but are made available to teachers for school-site assistance. Although anecdotal information provides evidence that proximity tends to increase use, it is still unclear whether, from the teachers' perspective, having a curriculum specialist resident on a school site is a substantially different resource from sharing the specialist across several schools. The question as asked on the NAEP background questionnaire does not allow for a distinction to be made regarding the use and usefulness of curriculum specialists residing on a school site as opposed to those who reside at the district office.

*Planning time* provided to teachers by districts and schools is another resource or form of support for the improvement of mathematics teaching and learning. Teachers in schools where classes are organized departmentally are typically provided with one or more periods of planning time: historically, these schools have been junior high and secondary schools. The provision of planning time during the school day is less typical for teachers of self-contained classrooms; generally, these classrooms are in elementary schools. Furthermore, any available planning time for teachers in self-contained classrooms would most likely be expected to cover the full curriculum rather than being designated for a specific subject area. As the data in Table 4.3 show, in 1996, 71 percent of fourth-grade students were taught by teachers who reported having more than three hours a week designated for preparation time. This was for overall preparation time not just for mathematics. The number of hours their teachers had designated for preparation was found not to be related to grade 4 students' performance on the 1996 NAEP mathematics assessment.

**Table 4.3**

**Percentage of Students and Average Scale Score by Teachers' Designated Preparation Time per Week, Grades 4 and 8, 1996**



		Hours of Designated Preparation Time							
		Less than One Hour		One to Two Hours		Three to Four Hours		Five or More Hours	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
	All Students	8	221	22	223	44	227	27	223
<b>Grade 8</b>									
	All Students	4	271	8	268	31	276	57	273

NOTE: Information in this table is for both public and nonpublic students. Row percentages may not sum to 100 percent due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress, 1996 Mathematics Assessment.

At the eighth-grade level, 57 percent of students were taught by teachers with five or more hours of preparation time per week. At this grade level also, the preparation time was intended for all classes or subjects taught, not just the mathematics class in which their students was enrolled. At the least, this averages out to about a typical 50-minute class period per day. Only 12 percent of students were taught mathematics by teachers who indicated having two hours or less of designated preparation time per week. As with fourth-grade students, eighth-grade students' performance on the 1996 NAEP mathematics assessment was found not to be related to their teachers' hours of designated preparation time.

## ***Availability of Calculators***

Calculators can be one of the most useful tools in the teaching and learning of mathematics. NCTM standards emphasize the importance of knowledge and skills in using calculators.<sup>4</sup> Tables 4.4 and 4.5 present information about students' access to calculators. At the fourth- and eighth-grade levels, the information is based on teachers' reports on students' access to school-owned calculators, while at the twelfth-grade level, the information is based on students' reports on the availability of calculators to do mathematics schoolwork. Further, the information on twelfth-grade students is specific to those who indicated they were currently enrolled in a mathematics class. In 1996, 64 percent of twelfth-grade students indicated that they were taking a mathematics class.<sup>5</sup>

---

<sup>4</sup> NCTM (1989). op. cit.

<sup>5</sup> NAEP, 1996 mathematics assessment.

Table 4.4

**Percentage of Student and Average Scale Score  
by Access to School-Owned Calculators,  
Grades 4 and 8**



		Have Access		Do Not Have Access	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>					
	1996	84†	226	16†	217
	1992	59	223	41	215
<b>Grade 8</b>					
	1996	80	273	20	279
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	82	263	18	267
	Pre-Algebra	79	270	21	274
	Algebra	77	296	23	301

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

In 1996, 84 percent of fourth-grade students had access to school-owned calculators in their mathematics class. This percentage was a significant increase over the 59 percent of students in 1992 who had access to school-owned calculators. Although access to calculators does not necessarily imply that calculators are used appropriately, the increase in availability implies a recognition of their potential usefulness in the teaching and learning of mathematics.<sup>6</sup>

A similar percentage (80%) of eighth-grade students also had access to school-owned calculators in their mathematics class in 1996. Furthermore, similar percentages of students had access to school-owned calculators in each of the three different types of mathematics classes, eighth-grade mathematics, pre-algebra, and algebra. No comparable data on calculator availability at grade 8 were available for 1992.

<sup>6</sup> More information about calculator use in classroom instruction is presented in a forthcoming report on the 1996 NAEP mathematics assessment about student work and instructional practices.

Table 4.5

**Percentage of Students and Average Scale Score by  
Access to Calculators for Mathematics Schoolwork,  
Grade 12 Enrolled in Mathematics**



	Have Access		Do Not Have Access	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Assessment Year				
1996	95†	313†	5†	283
1992	92	309	8	282

† Significantly different from 1992.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996, 1992 Mathematics Assessment.

Among twelfth-grade students who stated that they were taking mathematics classes in 1996, 95 percent reported that they had calculators available to do their mathematics schoolwork. The data do not indicate what proportion of these calculators were school-owned as opposed to student-owned. The percentage of students with access to calculators for mathematics schoolwork in 1996 was significantly higher than the 92 percent who reported this access in 1992. Further, twelfth-grade students with access to calculators in 1996 outperformed students who had such access in 1992 on the NAEP mathematics assessment.

Calculators that are accessible to students are becoming more sophisticated. No longer are students limited to 4-function calculators; many students have scientific and even graphing calculators available to them. In order for these resources to positively impact student learning, however, the key is appropriate use. Teachers must be able to teach conceptual understanding of mathematics while taking advantage of the more sophisticated capabilities of today's calculators. Students must be able to use calculators as tools to advance their mathematical understanding and not to compensate for lack of ability to engage in procedural computational tasks.

As shown in Table 4.6, 61 percent of eighth-grade students reported using scientific calculators; only 11 percent reported using graphing calculators. Perhaps as expected, a significantly higher percentage of eighth-grade students enrolled in algebra reported using scientific calculators (73%) than students in pre-algebra (59%) or eighth-grade mathematics (55%). The same was true for graphing calculators, where a significantly higher percentage of students in algebra (18%) reported using a graphing calculator compared to the percentages of students in pre-algebra (10%) or eighth-grade mathematics (7%). Therefore, although similar percentages of students in the different mathematics classes apparently have access to school-owned calculators (see Table 4.3), the use of scientific and graphing calculators is more common in the more advanced mathematics classes.

**Table 4.6**

***Percentage of Students and Average Scale Score by Use of Scientific and Graphing Calculators, Grades 8 and 12, 1996***



		Scientific Calculators		Graphing Calculators	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 8</b>					
All Students		61	277	11	275
<b>Students Enrolled in:</b>					
Eighth-Grade Mathematics		55	264	7	251
Pre-Algebra		59	271	10	270
Algebra		73	299	18	297
<b>Grade 12</b>					
Students Enrolled in Mathematics Course		72	310	62	321

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessment

Although students taking mathematics classes in their senior year of high school are distributed across all levels of mathematics courses from pre-algebra (or even lower levels) to calculus, higher-level courses predominate. It is therefore not unexpected that, in 1996, a significantly higher percentage of twelfth-grade students reported using scientific and graphing calculators (72% and 62%, respectively) than did eighth-grade students.

## Summary

Schools can support the teaching and learning of mathematics by providing needed resources and encouraging effective use of these resources. Resources, as described in this report, include not only curricular materials or instructional tools provided for classroom instruction but also time and support for teachers as they plan and implement classroom lessons. In 1996, the majority of fourth- and eighth-grade students had teachers of mathematics who reported receiving most of the instructional materials and other resources they needed. Although the performance of grade 4 students on the 1996 NAEP mathematics assessment was not related to the level of resources their teachers reported receiving, the performance of grade 8 students was. In particular, the more resources their teachers felt they were getting, the higher eighth-grade students' performance. Over time, the level of resources at grade 4 appears to have remained stable, while at grade 8 resource availability appears to have been increasing. Specifically, the percentage of students whose teachers in 1996 reported receiving some or none of the resources they needed (21%) was significantly lower than the percentage of students whose teachers reported that level in 1992 (33%).

In terms of individual teacher supports, 43 percent of fourth-grade students had teachers who indicated having access to curriculum specialists for help or advice in mathematics. A similar 49 percent of eighth-grade students were taught mathematics by teachers who reported having access to such curriculum specialists.

As perhaps would be expected in light of the way most of our nation's fourth and eighth grades are organized, teachers of eighth-grade students reported more overall designated preparation time than teachers of fourth-grade students. In 1996, only 27 percent of fourth-grade students were being taught mathematics by teachers with five or more hours per week of designated preparation time, while 57 percent of eighth-grade students were being taught by teachers with this much designated preparation time. For both grades 4 and 8, student performance on NAEP was not related to the amount of designated preparation time their teachers reported having.

In 1996, 84 percent of fourth-grade students were being taught by teachers who indicated that their students had access to school-owned calculators to do schoolwork. This was a significantly higher percentage than in 1992. A similar 80 percent of eighth-grade students were being taught by teachers who indicated that their students had access to school-owned calculators. The percentages of students in classes with access to calculators was the same regardless of the type of mathematics course (algebra, pre-algebra, and eighth-grade mathematics) in which students were enrolled. Almost all (95%) of twelfth-grade students taking a mathematics class reported having a calculator available to do mathematics schoolwork. This was somewhat higher than the 92 percent who reported such availability in 1992. Furthermore, students who in 1996 reported access to calculators outperformed students who in 1992 reported access to calculators.

As would be expected, the availability of scientific and graphing calculators increase with grade and level of course. In 1996, 61 percent of eighth-grade students reported using scientific calculators, and 11 percent reported using graphing calculators. A significantly higher percentage of eighth-grade students enrolled in algebra reported using scientific calculators than did students enrolled in pre-algebra or eighth-grade mathematics classes. This pattern also held for graphing calculators where a higher percentage of students in algebra reported their use than students in pre-algebra or eighth-grade mathematics classes. Twelfth-grade students taking mathematics appear to have had greater exposure than their younger peers to scientific and graphing calculators: 72 percent of twelfth-grade students taking mathematics in the twelfth grade reported using scientific calculators and 62 percent reported using graphing calculators.



## Chapter 5

# Conclusion

The purpose of this report was to provide information on various aspects of mathematics education to policymakers, school administrators, and others interested in the education of our nation's children. In particular, we summarized the characteristics of mathematics teachers in classrooms, the status of a variety of educational policies related to mathematics education, and the availability of some important resources that support the teaching and learning of mathematics. The focus was on information provided by students participating in the NAEP 1996 mathematics assessment, their teachers of mathematics, and their school administrators. We also reported, where available, corresponding trend data from the 1992 and 1990 administrations of NAEP in mathematics.

Interestingly, many of the patterns observed in this examination of the NAEP data appear to be corroborated by other data on the same or similar topics collected through other programs, particularly NCES' School and Staffing Survey (SASS).<sup>1</sup> Some points of correspondence with SASS will be summarized below. Before doing so, however, we point out a few methodological differences between the two surveys. First, in NAEP the unit of analysis is always the student, whereas in SASS the unit of analysis may be the teacher or the school. Second, the SASS data reported here were collected during the 1993–94 school year, while the NAEP data were collected in the 1995–96 school year. Third, NAEP data are limited to teachers of fourth-grade and eighth-grade students, while SASS data generally include all elementary or secondary grade teachers. In addition to these three differences, the national NAEP findings included in this report are based on combined public and nonpublic school data, while some of the SASS data reported below are based on public schools only or nonpublic schools only. Nevertheless, the comparisons are informative.

The 1996 NAEP and 1993–94 SASS data, as perhaps expected, both show that teachers with general education responsibilities tended to have college majors in general education, while fewer teachers whose main teaching assignment was mathematics tended to have a general education major only. The 1996 NAEP data indicated that 83 percent of fourth-grade students (who are primarily assigned to intact classrooms) and 32 percent of eighth-grade students (who more frequently have different teachers for different academic subjects) were being taught

---

<sup>1</sup> We did not go to the original sources for the SASS data reported here. We used what has been published in Henke, R. R., Choy, S., Chen, X., Geis, S., Alt, M. N., and Broughman, S. P., 1997, op. cit.

mathematics by teachers with an undergraduate or graduate major in education and not in mathematics or mathematics education. Correspondingly, the 1993–94 SASS data showed that 83 percent of public school teachers with a kindergarten or general education assignment had a major in general education, while 26 percent of public school teachers with a main assignment in mathematics or science had a major in general education rather than in a particular subject area or the teaching of it (for example, mathematics or mathematics education).

In general, both surveys also show that mathematics teaching certificates are more common among teachers of mathematics than having a college major in mathematics or mathematics education. In 1996, NAEP found that 32 percent of fourth-grade students were being taught mathematics by teachers with a mathematics teaching certificate, while 13 percent had teachers with an undergraduate or graduate degree in mathematics or mathematics education. At the eighth-grade level, 81 percent of students were being taught mathematics by a teacher with a mathematics teaching certificate, while 61 percent of students had teachers with an undergraduate or graduate degree in mathematics or mathematics education. In the 1993–94 SASS, 81 percent of teachers with a main assignment field in mathematics indicated that they had a mathematics teaching certificate.

Both NAEP and SASS found that teachers had considerable years of teaching experience. In the 1996 NAEP, the modal response of teachers of students at both grades 4 and 8 was 11–24 years of elementary and secondary teaching experience. SASS found that in 1993–94, the modal response of teachers was 20 years or more of teaching experience (the highest level of response provided), with an average of about 15 years.

NAEP and SASS both asked numerous questions about professional development activities, although the questions were less comparable than on other topics. Examining data from the two sources together, however, offers an interesting opportunity to study professional development. For example, SASS found that in 1993–94, among the various types of professional development activities, teachers were most likely to participate in school- or district-sponsored workshops or in-service programs: 92 percent of teachers reported that they had participated in these activities. NAEP found that 45 percent of eighth-grade students had teachers who participated in more than 15 hours of professional development activities that focused on mathematics or mathematics education — activities such as workshops and seminars, attendance at professional meetings and conferences, district-sponsored workshops, and external workshops. SASS also found that in 1993–94, about 40 percent of teachers indicated that they participated in college, university, extension or adult education courses since the end of the last school year. And, responses to the 1996 NAEP indicate that 21 percent of fourth-grade students and 26 percent of eighth-grade students had teachers who had taken one or more college courses in mathematics or mathematics education during the past two years.

SASS found that elementary school teachers in self-contained classrooms spent about 21 hours per week teaching core academic subjects. About 10 of those hours were spent teaching English, reading and language arts, and the rest of their teaching time was divided among arithmetic and mathematics, social studies and history, and science lessons. NAEP found that

68 percent of fourth-grade teachers indicated that they spent four or more hours per week on mathematics instruction. Although the data are less comparable because many eighth grades are departmentalized rather than self-contained classrooms, the NAEP data also show that 33 percent of eighth-grade students had teachers who spent four or more hours per week on mathematics instruction.

In 1993–94 SASS found that 73 percent of public school teachers and 86 percent of private school teachers agreed that necessary materials (such as textbooks, supplies, and a copy machine) were available to staff as needed. In the 1996 NAEP mathematics assessment, teachers of fourth- and eighth-grade students were asked to indicate the extent to which they felt they received the resources they needed to teach their class; 66 percent of fourth-grade students and 79 percent of eighth-grade students had teachers who indicated that they got most or all of the resources they needed.

Perhaps it goes without saying that reports such as this report on policies and practices create many more questions than they answer. What is encouraging is that, as shown above, data collected through other efforts such as SASS, can serve to complement and augment the data NAEP collects. Furthermore, the data examined and reported on in this report are only a subset of what are available through NAEP. Therefore, opportunities for further research and investigations are available and encouraged.

Finally, this report does not include information about the curriculum content of classroom instruction or the pedagogical practices utilized in classrooms. Neither does it detail students' course-taking patterns. These are the types of information that readers will logically want to know about in tandem with the policies and practices reported here. As mentioned previously, those types of information, as well as information about students' performance on cognitive mathematics questions, are included in companion reports in the series on the 1996 NAEP mathematics assessment.

# Appendix A

## ***Procedural Appendix***

### ***The NAEP 1996 Mathematics Assessment***

The 1996 assessment was the first update of the NAEP mathematics assessment framework since the release of the National Council of Teacher of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*.<sup>1</sup> This update sought to incorporate new knowledge about the teaching and learning of mathematics while also ensuring comparability of results across the 1990, 1992, and 1996 assessments.

### ***The Assessment Design***

Each student participating in the assessment received a booklet containing three 15-minute blocks of cognitive items. NAEP uses an adaptation of matrix sampling called balanced incomplete block (BIB) spiraling — a design that enables broad coverage of mathematics content while minimizing the burden for any one student. The balanced incomplete block part of the design assigns blocks of items to booklets; each pair of blocks appears together in at least one booklet, and each pair of booklets shared at least one block. The spiraling part of the design cycles the booklets for administration, so that typically only a few students in any assessment session receive the same booklet.

Of the 17 blocks in the national sample at grade 4, and of the 19 blocks in the national sample at each of grades 8 and 12, three at each grade were carried forward from the 1990 assessment, and five were carried forward from the 1992 assessment, to allow for the measurement of trends across time. The remaining blocks of questions at each grade level contained new questions that were developed for the 1996 assessment as specified by the updated framework.

Each cognitive block of mathematics items consisted of multiple-choice and constructed-response questions. In addition, five to seven of the blocks at each grade allowed for the use of calculators. At grade 4, students were provided four-function calculators, and at grades 8 and 12, students were provided scientific calculators. Prior to the assessment, all students were

---

<sup>1</sup> NCTM. (1989). op. cit.

trained to use these calculators. For several blocks, students were given manipulatives (including geometric shapes, three dimensional models, and spinners). For two of the blocks, students were given rulers at grade 4 and rulers and protractors at grades 8 and 12.

Each student booklet also contained three sets of student background questions. The first set included general questions about the student's race or ethnicity, mother's and father's level of education, number and type of reading materials in the home, amount of time spent on homework, and student's academic expectations. The second set was directed specifically at the student's mathematics background and included questions about mathematics instructional activities, mathematics courses taken, use of specialized resources such as calculators in mathematics classes, and views on the utility and value of mathematics. These first two sets of background questions preceded the cognitive blocks in the assessment. The third set of questions followed the cognitive question blocks and contained five questions about students' motivation to do well on the assessment, their perception of the difficulty of the assessment, and their familiarity with the types of cognitive questions included.

In addition to the student assessment booklets, three other instruments provided data relating to the assessment — a mathematics teacher questionnaire, a school characteristics and policy questionnaire, and a students with disabilities/limited English proficiency (SD/LEP) student questionnaire. The first two of these three additional instruments are especially relevant to this report.

The teacher questionnaires were administered to the mathematics teachers of each of the fourth- and eighth-grade students participating in the assessment. Because over a third of twelfth-grade students were not enrolled in mathematics, no questionnaires were administered to twelfth-grade mathematics teachers. The teacher questionnaire consisted of three sections. The first section focused on the teacher's general background and experience; the second section focused on the teacher's background related to mathematics; and the third section focused on classroom mathematics instruction. Because the sampling for the teacher questionnaire was based on participating students, the responses to the mathematics teacher questionnaire do not necessarily represent all fourth- or eighth-grade mathematics teachers in the nation or in a state. Rather, they represent teachers of the representative sample of students assessed. It is important to note that in this report, as in all NAEP reports, the student is always the unit of analysis, even when information from the teacher or school questionnaire is being reported. Using the student as the unit of analysis makes it possible to describe the educational context experienced by representative samples of students.

The school characteristics and policy questionnaire was given to the principal or other administrator in each participating school. The questions asked about the principal's background and experience; school policies, programs, and facilities; and the demographic characteristics and backgrounds of the students and teachers in that school.

The SD/LEP student questionnaires were completed by school staff for each student identified as (1) having an Individualized Education Plan (IEP), or (2) having limited English proficiency (LEP), regardless of whether the student participated in the assessment.


## ***National and State Samples***

The national results presented in this report are based on nationally representative probability samples of fourth-, eighth-, and twelfth-grade students. The samples were selected using a complex multistage sampling design that involved sampling students from selected schools within selected geographic areas across the country. The sample design had the following stages:

1. selection of geographic areas (a county, group of counties, or metropolitan statistical area);
2. selection of schools (public and nonpublic) within the selected areas; and
3. selection of students within selected schools.

Each selected school that participated in the assessment and each student assessed represents a portion of the population of interest. Sampling weights are needed to make valid inferences between the student samples and the respective populations from which they were drawn. Sampling weights account for disproportionate representation due to the oversampling of students who attend schools with high concentrations of Black and/or Hispanic students and who attend nonpublic schools. Sampling weights also account for lower sampling rates for very small schools.

Table A.1 provides a summary of the weighted and unweighted student sample sizes for the national mathematics assessment. The numbers reported include public and nonpublic school students.

<b>Table A.1</b>		<b><i>National School and Student Sample Sizes for the NAEP 1996 Mathematics Assessment</i></b>		THE NATION'S REPORT CARD 
	<b>Number of Schools</b>	<b>Unweighted Student Sample Size</b>	<b>Weighted Student Sample Size</b>	
Grade 4	281	6,627	3,714,998	
Grade 8	261	7,146	3,570,116	
Grade 12	264	6,904	2,830,443	

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.



Although students in public and nonpublic schools participated in the state assessment, the results of the 1996 state assessment program in mathematics provided in this report are based on state-level samples of fourth- and eighth-grade students from public schools only. The samples of both public and nonpublic school fourth- and eighth-grade students were selected based on a two-stage sample design that entailed selecting schools within participating jurisdictions and selecting students within schools. The first-stage samples of schools were selected with probability proportional to the fourth- or eighth-grade enrollment in those schools. Special procedures were used for jurisdiction that have many small schools and for jurisdictions that have a small number of schools. As with the national samples, the jurisdiction samples were weighted to allow for valid inferences about the populations of interest.

In carrying out the 1996 state assessment program, the National Center for Education Statistics, (NCES) established participation rate standards that jurisdictions were required to meet in order for their results to be reported. NCES also established additional standards that required the annotation of published results for jurisdictions whose sample participation rates were low enough to raise concerns about their representativeness.

No jurisdictions at grade 4 and three states at grade 8 (Nevada, New Hampshire, and New Jersey) failed to meet the initial public school participation rate of 70 percent. For these states, results for the eighth-grade public school students are not reported in this or any report of NAEP 1996 mathematics assessment findings. Several other jurisdictions whose results were published received a notation to indicate possible nonresponse bias.

To help ensure adequate sample representation for each jurisdiction participating in the 1996 state assessment program, NAEP provided substitutes for nonparticipating public and nonpublic schools. (When possible, a substitute school was provided for each initially selected school that declined participation.) For jurisdictions that used substitute schools, the assessment results were based on the student data from all schools participating from both the original sample and the list of substitutes (unless an initial school and its substitute eventually participated, in which case only the data from the initial school were used). For jurisdictions that did not use substitute schools, the participation rates were based on participating schools from the original sample.

NCES standards require weighted school participation rates before substitution of at least 85 percent to guard against potential bias due to school nonresponse. The NCES standards do not explicitly address the use of substitute schools to replace initially selected schools that declined to participate in the assessment. However, considerable technical consideration has been given to this issue. Even though the characteristics of the substitute schools were matched as closely as possible to the characteristics of the initially selected nonparticipation on initially selected schools, substitution does not entirely eliminate the possibility of bias because of the nonparticipation of initially selected schools. Thus, for the weighted school participation rates that included substitute schools, the guideline was set at 90 percent. This is expressed in the following guideline:

*A jurisdiction will receive a notation if its weighted participation rate for the initial sample of schools was below 85 percent AND the weighted school participation rate after substitution was below 90 percent.*



At grade 4, nine states did not meet this guideline for public schools: Arkansas, Iowa, Michigan, Montana, Nevada, New Jersey, New York, Pennsylvania, and Vermont. At grade 8, seven jurisdictions did not meet this guideline for public schools: Arkansas, Iowa, Michigan, Montana, New York, Vermont, and Wisconsin.

The NCES standards specify that attention should be given to the representativeness of the sample coverage. Thus, inadequate representation of an important segment of a jurisdiction's population is of concern, regardless of the overall participation rate. At grade 4, Alaska and South Carolina (for public schools) and at grade 8, Alaska, Maryland, and South Carolina (for public schools) failed to meet the following NCES guideline concerning strata-specific participation rates.

*A jurisdiction that is not already receiving a notation for problematic overall school or student participation rates will receive a notation if the sampled students within participating schools included a class of students with similar characteristics that had a weighted student response rate of below 80 percent, and from which the nonresponding students together accounted for more than five percent of the jurisdiction's weighted assessable student sample. Student groups from which a jurisdiction needed minimum levels of participation were determined by the age of the students, whether or not the student was classified as a student with disability (SD) or of limited English proficiency (LEP), and the type of assessment session (monitored or unmonitored). In addition, for public schools, classes of schools were determined by school level of urbanization, minority enrollment, and median household income of the area in which the school is located.*

This guideline addresses the concern that if nonparticipating schools were concentrated within a particular class of schools, the potential for substantial bias remained, even though the overall level of school participation appeared to be satisfactory. Nonresponse adjustment cells for schools were formed within each jurisdiction; the schools within each cell were similar in terms of minority enrollment, degree of urbanization, and/or median household income for public schools, and school type and location for nonpublic schools, as appropriate for each jurisdiction. If more than five percent (weighted) of the sample schools (after substitution) were nonparticipants from a single adjustment cell, then the potential for nonresponse bias was too great.

In one state (Alaska), the public school student participation rate for grade 8 fell below the NCES-prescribed criteria of 85 percent. No other notations related to student participation rates appear in NAEP 1996 mathematics reports. For a more detailed description of the sampling procedures see *NAEP 1996 Mathematics Report Card for the Nation and the States*.<sup>2</sup>

---

<sup>2</sup> Reese, C.M., Miller, K.E., Mazzeo, J., and Dossey, J. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

## ***Students with Disabilities (SD) and Limited English Proficient (LEP) Students***

It is NAEP's intent to assess all selected students. Therefore, every effort was made to ensure that all selected students who were capable of participating in the assessment were assessed.

However, some students sampled for participation in NAEP may be excluded from the sample on the basis of carefully defined criteria. These criteria are described in *NAEP 1996 Mathematics Report Card for the Nation and the States*.<sup>3</sup>

## ***Data Collection and Scoring***

As with all NAEP assessments, data collection was conducted by trained field staff. For the national assessment, this was accomplished by Westat, the NAEP contractor for data collection. For the state assessments, data were collected by local school personnel after training by Westat representatives. Materials collected as part of the 1996 assessment were shipped to National Computer Systems, where trained readers evaluated the responses to the constructed-response questions using scoring rubrics or guides prepared by the Educational Testing Service (ETS).

## ***Data Analysis and IRT Scaling***

Subsequent to the professional scoring, all information was transcribed to the NAEP database at ETS. Each processing activity was conducted with rigorous quality control. After the assessment information had been compiled in the database, the data were weighted according to the population structure. The weighting for the national and state samples reflected the probability of selection for each student as a result of the sampling design, adjusted for nonresponse. Through post-stratification, the weighting assured that the representation of certain subpopulations corresponded to figures from the U.S. Census and the Current Population Survey.<sup>4</sup>

Analyses were then conducted to determine the percentages of students who gave various responses to each cognitive and background question. Item response theory (IRT) was used to estimate average mathematics scale-score proficiency for the nation, various subgroups of interest within the nation, and for the states. IRT models the probability of answering an item correctly as a mathematical function of proficiency or skill. The main purpose of IRT analysis is to provide a common scale on which performance can be compared across groups, such as those defined by grades, and subgroups, such as those defined by gender or race/ethnicity. Because of the BIB spiraling design used by NAEP, students do not receive enough cognitive questions about a specific content area to provide reliable information about individual performance. Traditional test

---

<sup>3</sup> Ibid.

scores for individual students, even those based on IRT, would lead to misleading estimates of population characteristics, such as subgroup means and percentages of students at or above a certain proficiency level. Instead, NAEP constructs sets of plausible values designed to represent the distribution of proficiency in the population. A plausible value for an individual is not a scale score for that individual but may be regarded as a representative value from the distribution of potential scale scores for all students in the population with similar characteristics and identical patterns of item responses. Statistics describing performance on the NAEP proficiency scale are based on these plausible values. They estimate values that would have been obtained had individual proficiencies been observed — that is, had each student responded to a sufficient number of cognitive items so that proficiency could be precisely estimated.<sup>5</sup>

A separate score scale ranging from 0 to 500 was created to report performance for each content area (Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; Algebra and Functions). These scales summarize examinee performance across all three question types used in the assessment (multiple-choice, regular constructed-response, and extended-response). Each content area scale was based on the distribution of student performance across all three grades assessed in the 1996 national assessment (grades 4, 8, and 12) and had a mean of 250 and a standard deviation of 50. A composite score was also created as an overall measure of students' mathematics proficiency. The composite scale was a weighted average of the five content-area scales, where the weight for each content area was proportional to the relative importance assigned to the content areas in the specifications developed by the Mathematics Objectives Panel. The average mathematics composite scale score is the average scale score used in this report.

The NAEP proficiency scales make it possible to examine relationships between students' performance and a variety of background factors measured by NAEP. The fact that a relationship exists between achievement and another variable, however, does not reveal the underlying cause of the relationship, which may be influenced by a number of other variables. Similarly, the reported relationships do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

Most of the data analyses were conducted by ETS. However, some of the results presented in this report are based on additional analyses conducted by the American Institutes for Research using data sets provided by ETS.

More detailed information about data analysis and item response theory are presented in the *1996 NAEP Technical Report*.<sup>6</sup>

---

<sup>4</sup> For additional information about the use of weighting procedures in NAEP, see Johnson, E.G. (1989, December). *Journal of Education Statistics*, 14 (4), pp. 303-334.

<sup>5</sup> For theoretical justification of the procedures employed, see Mislevy, R. J. (1988). Randomization-based inferences about latent variables from complex samples. *Psychometrika*, 56 (2), pp. 177-196.

<sup>6</sup> Allen, N.J., Jenkins, F. Kulick, E., & Zelnick, C.A. (1997). *Technical report of the NAEP 1996 state assessment program in mathematics*. Washington, DC: National Center for Education Statistics.

## **Reporting Groups**

In this report, some of the results for eighth- and twelfth-grade students are also provided for separate subpopulations of students defined by the types of mathematics course they were currently or had previously taken. In any given analysis, however, results are only reported for subpopulations represented by sufficient numbers of students and adequate school distributions. For public school students, the minimum requirement is at least 62 students in a particular subgroup from at least five primary sampling units (PSUs).<sup>7</sup> For nonpublic school students, the minimum requirement is 62 students from at least five PSUs for the national assessment. Regardless of whether the subgroup was reported separately, the data for all students were included in computing overall results. Definitions of the course-taking subpopulations used in this report are presented below.

### ***Eighth-grade course taking***

Eighth-grade students responded to a question about what mathematics course they were currently taking. Students were provided with seven response options that included the following:

- I am not taking mathematics this year;
- Eighth-grade mathematics;
- Pre-algebra;
- Algebra;
- Integrated or sequential mathematics;
- Applied mathematics (technical preparation); and
- Other mathematics class.

The course-taking grouping variable used in this report is based on the subset of students who responded that they were taking eighth-grade mathematics, pre-algebra, or algebra. Students who marked some other response are not included in the subpopulation analysis.

### ***Twelfth-grade highest algebra-calculus course taken***

At the twelfth-grade level, the course-taking subpopulations are based on the highest level mathematics course students reported having taken in an algebra-calculus sequence. Students' responses were edited for consistency with the standard course-taking sequence. That is, the student was not credited as having taken a given course unless his or her responses also indicated completion of the course prerequisites.

---

<sup>7</sup> For the national assessment, a PSU is a selected geographic region (a county, a group of counties, or metropolitan statistical areas). For the state assessment program, a PSU is most often a single school.

The twelfth-grade grouping variable has six categories:

1. Not Taken Pre-Algebra: These are students who had less than a year of introduction to algebra or pre-algebra.
2. Pre-Algebra: These are students who had a year or more of introduction to algebra or pre-algebra, but not first-year algebra.
3. First-Year Algebra: These are students who had a year or more of first-year algebra, but not second-year algebra.
4. Second-Year Algebra: These are students who had a year or more of second-year algebra, but not precalculus.
5. Pre-Calculus: These are students who had a year or more of precalculus, third-year algebra, elementary functions or analysis, but not calculus.
6. Calculus: These are students who had a year or more of calculus.

## ***Guidelines for Analysis and Reporting***

This report describes students', teachers', and principals' responses to background questions as well as mathematics performance for fourth-, eighth-, and twelfth-grade students. The report also compares the performance results for various groups of students within these populations (e.g., subgroups formed of those who responded to a specific background question in a particular way or in accordance with the individual course-taking groups described above). However, it does not include an analysis of the relationships among combinations of these subpopulations or background questions.

### ***Estimating variability***

The statistics presented in this report are estimates of group and subgroup performance based on samples of students, and they therefore differ from the statistics that could be calculated if every student in the nation answered every question. The degree of uncertainty associated with these sample-based estimates should, therefore, be taken into account. Two components of uncertainty are accounted for in the variability statistics based on student ability: (1) the uncertainty due to sampling only a relatively small number of students, and (2) the uncertainty due to sampling only a relatively small number of cognitive questions per student. The first component alone accounts for the variability associated with the estimated percentages of students who had certain background characteristics or who answered a certain cognitive question correctly.

Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate. NAEP uses a jackknife replication procedure to estimate standard errors. The jackknife standard error provides a reasonable measure of uncertainty for any student information that can be observed without error. However, because each student typically responds to only a few questions within any

content strand, the scale score for any single student would be imprecise. In this case, plausible values technology can be used to describe the performance of groups or subgroups of students, but the underlying imprecision involved in this step adds another component of variability to statistics based on NAEP scale scores.<sup>8</sup>

Typically, when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard error may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated.

The reader is reminded that, like findings from all surveys, NAEP results are subject to other kinds of error, including the effects of imperfect adjustments for student and school nonresponse and unknowable effects associated with the particular instrumentation and data collection methods. Nonsampling errors can be attributed to a number of sources — inability to obtain complete information about all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain questions); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors in collecting, processing, sampling, and estimating missing data. The extent of nonsampling error is difficult to estimate, and because of their nature, the impact of such errors cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

### ***Drawing inferences from the results***

As noted, the percentages of students and average scale scores used in reporting NAEP results are based on samples rather than on the entire population of fourth-, eighth-, or twelfth-graders in the nation or a jurisdiction. Consequently, the numbers reported are estimates and are subject to a measure of uncertainty, reflected in the standard error of the estimate. When the percentages or average scale scores of certain groups are compared, the standard error should be taken into account, and observed similarities or differences should not be solely relied on. Therefore, the comparisons discussed in this report are based on statistical tests that consider the standard errors of those statistics as well as the magnitude of the differences among the averages or percentages.

---

<sup>8</sup> For more details, see Johnson, E.G. & Rust, K.F. (1992). Population inferences and variance estimation for NAEP data. *Journal of Educational Statistics*, 17(2), pp. 175-190.



The results from the sample, taking into account the uncertainty associated with all samples, are used to make inferences about the population. Using confidence intervals based on the standard errors provides a way to make inferences about the population averages and percentages in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample average scale score  $\pm 2$  standard errors approximates a 95 percent confidence interval for the corresponding population quantity. This statement means that one can conclude with approximately a five percent level of significance that the average performance of the entire population of interest (e.g., all fourth-grade students in public schools in a jurisdiction) is within  $\pm 2$  standard errors of the sample average.

As an example, suppose that the average mathematics scale score of the students in a particular group was 256 with a standard error of 1.2. A 95 percent confidence interval for the population quantity could be described in any of the following ways:

Average  $\pm 2$  standard errors  
 $256 \pm 2 \times 1.2$   
 $256 \pm 2.4$   
253.6, 258.4

Thus, one can conclude with a five percent level of significance that the average scale score for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, if the percentages are not extremely large or extremely small. For extreme percentages, confidence intervals constructed in the above manner may not be appropriate, and accurate confidence intervals can be constructed only by using procedures that are quite complicated.

Extreme percentages, defined by both the magnitude of the percentage and the size of the sample from which it was derived, should be interpreted with caution. The *NAEP 1996 Technical Report* contains a more complete discussion of extreme percentages.<sup>9</sup>

### **Analyzing group differences in averages and percentages**

Statistical tests are used to determine whether the evidence, based on the data from the groups in the sample, is strong enough to conclude that the averages or percentages are actually different for those groups in the population. If the evidence is strong (i.e., the difference is statistically significant), the report describes the group averages or percentages as being different (e.g., one group performed higher than or lower than another group), regardless of whether the sample averages or percentages appear to be approximately the same. If the evidence is not sufficiently strong (i.e., the difference is not statistically significant), the averages or percentages are described as being not significantly different, regardless of whether the sample averages or percentages appear to be approximately the same or widely discrepant.

---

<sup>9</sup> Allen, N.J., Jenkins, F., Kulick, E., & Zelnick, C.A. (1997). op. cit.



The reader is cautioned to rely on the results of the statistical tests rather than on the apparent magnitude of the difference between sample averages or percentages when determining whether the sample differences are likely to represent actual differences among the groups in the population.

To determine whether a real difference exists between the average scale scores (or percentages of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the averages (or percentages) of these groups for the sample. This estimate of the degree of uncertainty, called the standard error of the difference between two independent groups, is obtained by taking the square of each group's standard error, summing the squared standard errors, and taking the square root of that sum.

$$\text{Standard Error of the Difference} = SE_{A-B} = \sqrt{SE_A^2 + SE_B^2}$$

Similar to how the standard error for an individual group average or percentage is used, the standard error of the difference can be used to help determine whether differences among groups in the population are real. The difference between the averages or percentages of the two groups  $\pm 2$  standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim that the real difference between the groups is statistically significant (different) at the five percent level. In this report, differences among groups that involve poorly defined variability estimates or extreme percentages are not discussed.

As an example, to determine whether the average mathematics scale score of Group A is higher than that of Group B, suppose that the sample estimates of the average scale score and standard errors were as follows:

<u>Group</u>	<u>Average Scale Score</u>	<u>Standard Error</u>
A	218	0.9
B	216	1.1

The difference between the estimates of the average scale scores of Groups A and B is two points (218–216). The standard error of this difference is:

$$(0.9^2 + 1.1^2) = 1.4$$

Thus, an approximate 95 percent confidence interval for this difference is:

$$\begin{aligned} &\text{Difference } \pm 2 \text{ standard errors of the difference} \\ &2 \pm (2 \times 1.4) \\ &2 \pm 2.8 \\ &-0.8, 4.8 \end{aligned}$$

The value zero is within the confidence interval, therefore, there is insufficient evidence to claim that Group A outperformed Group B.

The procedures described in this section and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. However, in Chapters 2–4 of this report, many different groups are being compared (i.e., multiple sets of confidence intervals are being analyzed). In sets of confidence intervals, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. To hold the significance level for the set of comparisons at a particular level (e.g., 0.05), adjustments (called multiple comparison procedures) must be made to the methods described in the previous section. One such procedure, the Bonferroni method, was used in the analyses described in this report to establish confidence intervals for the differences among groups when sets of comparisons were considered.<sup>10</sup> Thus, the confidence intervals for the sets of comparisons in the text are more conservative than those described on the previous pages.

Most of the multiple comparisons in this report pertain to relatively small sets or families of comparisons. For example, for discussions concerning comparisons of eighth-grade course taken groups, three comparisons were conducted — all pairs of the three course taken groups. In these situations, Bonferroni procedures were appropriate. However, for the cross-state comparisons with a large family of comparisons, the False Discovery Rate (FDR) procedure was used to control the certainty level.<sup>11</sup>

Unlike Bonferroni procedures which controls the familywise error rate (i.e., the probability of making even one false rejection in the set of comparisons), the FDR procedure controls the expected proportion of falsely rejected hypotheses. Furthermore, Bonferroni procedures are considered conservative for large families of comparisons.<sup>12</sup> Therefore, the FDR procedure is more suitable for cross-state comparisons. A detailed description of the Bonferroni and FDR procedures appears in *NAEP 1996 Technical Report*.<sup>13</sup>

---

<sup>10</sup> Miller, R.G. (1996). *Simultaneous statistical inference*. New York: Wiley.

<sup>11</sup> Benjamin and Hochberg. (1995). False discovery rate (FDR) procedure. *Journal of the Royal Statistical Society, Series B*, No. 1, pp. 289-300.

<sup>12</sup> Williams, V.S., Jones, L.V., & Tukey, J.W. (1994). *Controlling error in multiple comparisons with special attention to the National Assessment of Educational Progress*. Research Triangle Park, NC: National Institute of Statistical Sciences.

<sup>13</sup> Allen, N.J., Jenkins, F., Kulick, E., & Zelnick, C.A. (1997). op. cit.

## ***Revisions to the NAEP 1990 and 1992 Mathematics Findings***

After the NAEP 1994 assessment was conducted, a technical problem was discovered in the procedures used to develop the NAEP mathematics scale for the 1992 mathematics assessment. This error affected the mathematics scale scores reported in 1992. The technical error has been corrected and the revised national and state scale score results for 1992 are presented in the NAEP 1996 mathematics reports. The technical problem is described in greater detail in the *NAEP 1996 Technical Report*.<sup>14</sup> A brief summary of the problem is presented in the *NAEP 1996 Mathematics Report Card of the Nation and the States*.<sup>15</sup>

---

<sup>14</sup> Ibid.

<sup>15</sup> Reese, C.M., Miller, K.E., & Mazzeo, J. (1997). op. cit., pp. 403-415

## Appendix B

### ***Standard Error Tables***

The comparisons presented in this report are based on statistical tests that consider the magnitude of the difference between group averages or percentages and the standard errors of those statistics. The following appendix contains the standard errors for the averages and percentages discussed in Chapters 2, 3, and 4.

#### ***Chapter 2***

Table B2.1	Standard Errors for Percentage of Students and Average Scale Score by Teachers' College Major, Grades 4 and 8
Figure B2.1	Standard Errors for Percentage of Students Whose Teachers Have a College Major in Mathematics, for the Nation and States: Public Schools Only, 1996
Table B2.2	Standard Errors for Percentage of Students by Teachers' Reports on One or More College Mathematics Courses Taken, Grades 4 and 8
Table B2.3	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Teaching Certification, Grades 4 and 8, 1996
Figure B2.2	Standard Errors for Percentage of Students Whose Teachers Have Mathematics Teaching Certificates, for the Nation and States: Public Schools Only, 1996
Table B2.4	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Years General Teaching Experience, Grades 4 and 8
Table B2.5	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Years of Mathematics Teaching Experience, Grades 4 and 8
Figure B2.3	Standard Errors for Percentage of Students Whose Teachers Report More Than Ten Years of Teaching Mathematics, for the Nation and the States: Public Schools Only, 1996

Table B2.6	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Hours of Professional Development During the Last Year, Grades 4 and 8, 1996
Figure B2.4	Standard Errors for Percentage of Grade 4 Students Whose Teachers Report More than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996
Figure B2.5	Standard Errors for Percentage of Grade 8 Students Whose Teachers Report More than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996
Table B2.7	Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Use of Technology, Grades 4 and 8, 1996
Figure B2.6	Standard Errors for Percentage of Grade 4 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and States: Public Schools Only, 1996
Figure B2.7	Standard Errors for Percentage of Grade 8 Students Whose Teachers Had Professional Development in Use of Technology, for the Nation and the States: Public Schools Only, 1996
Table B2.8	Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, Grades 4 and 8, 1996
Figure B2.8	Standard Errors of Percentage of Grade 4 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996
Figure B2.9	Standard Errors for Percentage of Grade 8 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996
Table B2.9	Standard Errors for Percentage of Students and Average Scale Score by Number of College Courses in Mathematics or Mathematics Education Teachers Have Taken During the Last Two Years, Grades 4 and 8, 1996
Table B2.10	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Knowledge of NCTM Curriculum and Evaluation Standards, Grades 4 and 8, 1996

### **Chapter 3**

Figure B3.1	Standard Errors for Percentage of Twelfth-Grade Students by Mathematics Graduation Requirement (Grades 9 through 12), 1996
Table B3.1	Standard Errors for Percentage of Twelfth-Grade and Average Scale Score by Highest Algebra-Calculus Course Taken and Mathematics Graduation Requirements (Grades 9 through 12), 1996

Table B3.2	Standard Errors for Percentage of Twelfth-Grade Students Who Have Taken Geometry by Mathematics Graduation Requirement (Grades 9 through 12), 1996
Table B3.3	Standard Errors for Percentage of Twelfth-Grade Students and Average Scale Score by Whether Specific Advanced Mathematics Courses of One Semester in Length Taught in Their School
Figure B3.2	Standard Errors for Percentage of Students Whose Schools Offer Algebra for High School Credit for Placement
Figure B3.3	Standard Errors for Percentage of Students in Schools That Offer Algebra for Eighth-Grade Students, for the Nation and States: Public Schools Only, 1996
Table B3.4	Standard Errors for Percentage of Eighth-Grade Students by Mathematics Course Enrollment and Availability of Algebra, 1996
Figure B3.4	Standard Errors for Percentage of Students Enrolled in Algebra, for the Nation and the States: Public Schools Only, 1996
Table B3.5	Standard Errors for Percentage of Students and Average Scale Score by Time on Mathematics Instruction, Grades 4 and 8
Figure B3.5	Standard Errors for Percentage of Grade 4 Students with Four or More Hours per Week of Mathematics Instructions, for the Nation and States: Public Schools Only, 1996
Figure B3.6	Standard Errors for Percentage of Grade 8 Students with Four or More Hours per Week of Mathematics Instructions, for the Nation and States: Public Schools Only, 1996

## **Chapter 4**

Table B4.1	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Reports on the Availability of Resources, Grades 4 and 8
Figure B4.2	Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Have Access to Mathematics Curriculum Specialists, Grades 4 and 8
Table B4.3	Standard Errors for Percentage of Students and Average Scale Score by Teachers' Designated Preparation Time per Week, Grades 4 and 8, 1996
Table B4.4	Standard Errors for Percentage of Students and Average Scale Score of Students by Access to School-Owned Calculators, Grades 4 and 8
Table B4.5	Standard Errors for Percentage of Students and Average Scale Score by Access to Calculators for Mathematics Schoolwork, Grade 12 Enrolled in Mathematics
Table B4.6	Standard Errors for Percentage of Students and Average Scale Score by Use of Scientific and Graphing Calculators, Grades 8 and 12, 1996

Table B2.1

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' College Major, Grades 4 and 8**



Undergraduate or Graduate Major									
		Mathematics		Mathematics Education but not Mathematics		Education but not Mathematics or Mathematics Education		Other	
Assessment Year		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4:									
All Students	1996	1.6	3.8	0.8	6.0	1.9	1.0	0.8	6.1
	1992	1.1	3.5	0.8	4.2	1.5	0.9	0.5	3.4
Grade 8:									
All Students	1996	2.8	1.8	1.7	2.5	2.8	2.5	1.2	3.6
	1992	2.7	1.5	1.3	2.9	2.6	2.1	1.1	2.6
Students Enrolled in:									
Eighth-Grade Mathematics	1996	4.3	2.4	2.5	4.4	3.9	2.5	1.2	3.9
	1992	3.0	1.7	1.7	3.0	2.8	2.2	1.4	3.3
Pre-Algebra	1996	4.2	2.4	2.9	3.1	3.4	2.7	1.9	4.6
	1992	4.2	1.6	1.5	3.5	4.6	3.7	1.9	3.2
Algebra	1996	4.4	2.1	1.9	3.8	4.2	3.8	1.5	6.7
	1992	4.0	2.3	2.4	3.1	3.3	3.8	1.8	7.1

NOTE: Information in this table is for both public and nonpublic school students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

Figure B2.1

**Standard Errors for Percentage of Students Whose Teachers  
Have a College Major in Mathematics, for the Nation and  
States: Public Schools Only, 1996**



<b>Grade 8</b>	
<b>Nation</b>	<b>3.0</b>
<b>Percent Above the National Average</b>	
District of Columbia	0.9
Massachusetts	3.6
Minnesota	3.0
Nebraska	2.9
New York	3.6
Rhode Island	1.0
Texas	3.2
Wyoming	1.2
<b>Percent Does Not Differ from the National Average</b>	
Alabama	3.8
Alaska	3.2
Arkansas	4.9
California	3.7
Colorado	3.5
Connecticut	3.5
Florida	2.8
Hawaii	1.2
Indiana	4.7
Iowa	4.7
Kentucky	3.8
Maine	4.0
Maryland	3.8
Michigan	4.4
Mississippi	4.0
Missouri	3.9
Montana	3.3
New Mexico	2.2
North Carolina	3.3
North Dakota	2.9
South Carolina	3.7
Utah	2.0
Vermont	2.3
Virginia	3.3
West Virginia	3.6
Wisconsin	4.8
DODDS	0.8
<b>Percent Below the National Average</b>	
Arizona	4.1
Delaware	1.1
Georgia	3.2
Louisiana	4.0
Oregon	3.8
Tennessee	4.1
Washington	3.7
Guam	1.3

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.



Table B2.2

**Standard Errors for Percentage of Students by Teachers' Reports on One or More College Mathematics Courses Taken, Grades 4 and 8**

THE NATION'S  
REPORT  
CARD



	Mathematics Course Content Area								
	Assessment Year	Teaching Methods	Number Systems and Numeration	Measurement	Geometry	College Algebra	Probability/Statistics	Calculus	Abstract/Linear Algebra
<b>Grade 4</b>									
All Students	1996	1.7	2.1	2.1	2.3	2.5	2.2	1.5	1.5
	1992	0.8	2.0	2.0	1.9	NA	2.3	1.1	2.0
<b>Grade 8</b>									
All Students	1996	3.0	3.2	3.4	2.5	2.2	2.7	2.3	2.7
	1992	2.2	2.1	2.5	2.3	NA	2.3	2.5	2.2
<b>Students Enrolled in:</b>									
Eighth-Grade Mathematics	1996	4.0	4.5	4.1	4.3	2.9	4.5	4.0	4.4
	1992	2.7	2.8	2.9	2.6	2.8	2.9	2.7	2.5
Pre-Algebra	1996	4.7	4.1	3.8	3.3	3.2	3.0	3.7	3.9
	1992	2.6	3.2	3.8	3.6	4.6	3.1	4.6	3.4
Algebra	1996	4.5	4.2	4.8	4.4	3.3	3.9	3.4	3.4
	1992	3.3	3.1	3.6	3.4	3.3	3.5	3.2	3.0

NOTE: Information in this table is for both public and nonpublic school students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

Table B2.3

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Teaching Certification, Grades 4 and 8, 1996**



	Type of Teaching Certification					
	Mathematics		Education but not Mathematics		Other	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>						
All Students	2.7	2.0	2.7	1.2	---	3.7
<b>Grade 8</b>						
All Students	2.3	1.3	2.3	3.6	---	7.1
<b>Students Enrolled in:</b>						
Eighth-Grade Mathematics	2.9	1.9	2.9	3.2	---	6.7
Pre-Algebra	3.1	1.8	3.0	3.4	---	***
Algebra	3.2	1.9	3.1	7.8	---	***

\*\*\*Sample size is insufficient to permit a reliable estimate.

--- Standard error estimates cannot be accurately determined.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

**Figure B2.2**

**Standard Errors for Percentage of Students Whose Teachers  
Have Mathematics Teaching Certificates, for the Nation and  
States: Public Schools Only, 1996**



<b>Grade 8</b>	
<b>Nation</b>	<b>2.2</b>
<b>Percent Above the National Average</b>	
Arkansas	2.8
Florida	1.2
Indiana	0.9
Maryland	1.5
Minnesota	0.7
Missouri	1.8
Nebraska	0.9
New York	1.9
North Carolina	1.6
North Dakota	1.4
Rhode Island	0.5
Texas	1.9
Utah	1.2
Vermont	1.1
Virginia	1.2
West Virginia	1.0
Guam	1.1
DODDS	0.6
<b>Percent Does Not Differ from the National Average</b>	
Alabama	2.5
California	2.8
Connecticut	2.7
Delaware	0.9
District of Columbia	0.8
Hawaii	1.2
Iowa	3.7
Kentucky	3.1
Massachusetts	3.2
Michigan	3.2
Montana	2.7
South Carolina	3.0
Washington	3.2
Wyoming	0.6
DDESS	0.8
<b>Percent Below the National Average</b>	
Alaska	3.2
Arizona	3.1
Colorado	3.1
Georgia	3.2
Louisiana	4.0
Maine	3.9
Mississippi	3.5
New Mexico	2.0
Oregon	3.0
Tennessee	3.6
Wisconsin	3.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B2.4

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Years General Teaching Experience, Grades 4 and 8**



		Years of Elementary or Secondary Teaching Experience							
		Five Years or Less		Six-Ten Years		Eleven-24 Years		25 Years or More	
Assessment Year		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
All Students	1996	1.8	1.8	1.8	1.6	2.3	1.6	2.2	2.2
	1992	1.8	1.9	1.5	1.9	2.3	1.2	1.4	1.9
<b>Grade 8</b>									
All Students	1996	2.7	2.0	2.2	2.4	3.6	1.8	2.8	3.8
	1992	1.6	2.1	1.3	2.6	1.9	1.3	1.6	2.2
<b>Students Enrolled in:</b>									
Eighth-Grade Mathematics	1996	3.1	2.2	3.4	3.9	5.1	2.7	3.3	3.5
	1992	2.0	2.0	1.7	2.6	2.4	2.1	2.2	2.0
Pre-Algebra	1996	4.4	3.1	3.0	3.7	4.4	2.8	4.2	2.8
	1992	2.3	2.7	2.0	4.3	2.6	1.5	1.8	2.6
Algebra	1996	2.8	4.4	2.7	3.9	4.9	3.2	5.2	4.1
	1992	2.5	4.0	2.3	3.9	3.4	2.7	2.9	3.2

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

**Table B2.5**

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Years of Mathematics Teaching Experience, Grades 4 and 8**



		Years of Mathematics Teaching Experience							
		Five Years or Less		Six-Ten Years		Eleven-24 Years		25 Years or More	
	Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
All Students	1996	2.2	1.7	1.9	1.6	2.5	1.3	1.9	2.5
<b>Grade 8</b>									
All Students	1996	2.8	1.5	2.4	2.8	3.5	1.8	2.5	4.3
	1992	1.8	1.7	1.6	2.2	2.2	1.5	1.7	2.9
<b>Students Enrolled in:</b>									
Eighth-Grade Mathematics	1996	4.0	2.6	3.6	3.8	5.2	2.8	2.9	4.6
	1992	2.3	1.7	1.8	2.4	2.7	2.2	2.0	2.6
Pre-Algebra	1996	4.0	2.7	2.9	3.5	4.3	2.3	3.9	3.4
	1992	2.9	2.5	2.7	3.5	3.3	1.6	2.0	3.2
Algebra	1996	2.7	3.2	2.9	3.5	4.0	3.0	4.0	5.1
	1992	2.5	3.6	2.7	3.5	4.0	2.9	3.1	3.0

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

**Figure B2.3**

**Standard Errors for Percentage of Students Whose Teachers Report More Than Ten Years of Teaching Mathematics, for the Nation and the States: Public Schools Only, 1996**



<b>Grade 8</b>	
<b>Nation</b>	<b>3.2</b>
<b>Percent Above the National Average</b>	
Connecticut	2.9
District of Columbia	1.0
Massachusetts	3.0
Rhode Island	0.9
West Virginia	2.6
DDESS	1.8
<b>Percent Does Not Differ from the National Average</b>	
Alabama	3.9
Arkansas	4.5
California	3.1
Florida	3.7
Georgia	3.7
Indiana	5.0
Iowa	4.1
Louisiana	4.0
Maine	3.9
Maryland	4.3
Michigan	4.2
Minnesota	3.9
Mississippi	3.7
Missouri	3.8
Montana	3.1
Nebraska	3.1
New York	4.3
North Carolina	3.2
North Dakota	2.9
Oregon	3.6
South Carolina	4.1
Tennessee	4.2
Texas	3.4
Vermont	3.1
Virginia	3.5
Washington	3.4
Wisconsin	5.1
Wyoming	1.4
DODDS	0.9
<b>Percent Below the National Average</b>	
Alaska	2.3
Arizona	3.9
Colorado	3.6
Delaware	1.3
Hawaii	1.1
Kentucky	3.9
New Mexico	2.6
Utah	1.5
Guam	1.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B2.6

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Hours of Professional Development During the Last Year, Grades 4 and 8, 1996**



		Hours of Professional Development					
		Less than 6 Hours		Between 6 and 15 Hours		More than 15 Hours	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>							
	All Students	2.6	1.5	2.2	1.8	2.4	1.8
<b>Grade 8:</b>							
	All Students	3.1	3.0	2.4	1.8	3.2	1.8
<b>Students Enrolled in:</b>							
	Eighth-Grade Mathematics	4.3	4.3	3.2	2.2	4.4	1.9
	Pre-Algebra	4.3	3.4	3.6	2.4	4.6	2.8
	Algebra	4.4	4.1	3.8	3.3	3.7	2.5

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.



**Figure B2.4**

**Standard Errors for Percentage of Grade 4 Students Whose Teachers Report More than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996**



<b>Grade 4</b>	
<b>Nation</b>	<b>2.6</b>
<b>Percent Above the National Average</b>	
Arkansas	4.8
California	3.8
Massachusetts	3.2
Nevada	3.3
Texas	3.9
Vermont	3.6
<b>Percent Does Not Differ from the National Average</b>	
Alabama	2.8
Alaska	3.1
Arizona	2.3
Colorado	2.5
Connecticut	2.3
Delaware	0.9
District of Columbia	0.6
Florida	2.5
Georgia	2.6
Hawaii	2.6
Kentucky	3.6
Louisiana	3.4
Maine	3.6
Maryland	3.4
Michigan	3.3
Minnesota	2.7
Mississippi	3.7
Missouri	3.9
Montana	3.4
Nebraska	2.3
New Jersey	3.3
New Mexico	3.1
New York	2.2
North Carolina	2.7
North Dakota	3.3
Oregon	3.0
Rhode Island	2.6
South Carolina	3.2
Tennessee	2.6
Utah	2.9
Virginia	3.1
Washington	3.2
West Virginia	3.2
DODDS	1.6
<b>Percent Below the National Average</b>	
Indiana	2.3
Iowa	2.9
Pennsylvania	2.7
Wisconsin	2.3
Wyoming	2.6
Guam	0.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

School Policies and Practices Affecting Instruction in Mathematics 112

**B-13**

Figure B2.5

**Standard Errors for Percentage of Grade 8 Students Whose Teachers Report More than 15 Hours Professional Development in Mathematics or Mathematics Education, for the Nation and States: Public Schools Only, 1996**



Grade 8	
Nation	3.5
<b>Percent Above the National Average</b>	
California	3.7
District of Columbia	1.2
Florida	3.3
Kentucky	4.0
Massachusetts	3.8
Texas	3.8
<b>Percent Does Not Differ from the National Average</b>	
Alabama	4.1
Arizona	3.7
Arkansas	5.5
Colorado	3.9
Connecticut	3.5
Delaware	1.0
Georgia	2.8
Hawaii	1.1
Iowa	4.9
Louisiana	3.8
Maine	2.7
Maryland	3.8
Michigan	4.5
Minnesota	3.6
Mississippi	3.6
Missouri	3.9
Montana	3.6
Nebraska	3.2
New York	4.5
North Carolina	3.2
North Dakota	2.7
Oregon	3.9
South Carolina	3.7
Tennessee	3.6
Utah	2.2
Vermont	2.3
Virginia	3.3
Washington	3.8
West Virginia	3.5
Wisconsin	5.0
DDESS	1.7
DODDS	1.0
<b>Percent Below the National Average</b>	
Alaska	2.4
Indiana	3.7
New Mexico	2.3
Rhode Island	0.9
Wyoming	0.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

**Table B2.7**

**Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Use of Technology, Grades 4 and 8, 1996**



		<b>Teacher Had Professional Development in Past Five Years</b>			
		<b>Yes</b>		<b>No</b>	
		<b>Percentage of Students</b>	<b>Average Scale Score</b>	<b>Percentage of Students</b>	<b>Average Scale Score</b>
<b>Grade 4.</b>					
	All Students	1.8	1.0	1.8	2.6
<b>Grade 8</b>					
	All Students	2.6	1.5	2.6	2.2
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	4.2	1.9	4.2	2.8
	Pre-Algebra	3.7	2.0	3.7	3.5
	Algebra	2.6	2.0	2.6	2.7

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Figure B2.6

**Standard Errors for Percentage of Grade 4 Students Whose Teachers  
Had Professional Development in Use of Technology, for the Nation  
and States: Public Schools Only, 1996**



Grade 4	
Nation	1.8
<b>Percent Above the National Average</b>	
Florida	1.5
Iowa	2.4
Kentucky	1.3
North Carolina	2.2
North Dakota	2.2
Utah	1.6
West Virginia	2.4
Wyoming	1.9
DOEDS	0.6
<b>Percent Does Not Differ from the National Average</b>	
Alabama	2.8
Alaska	2.3
Arizona	2.8
California	2.5
Colorado	2.0
Connecticut	2.6
Georgia	2.0
Hawaii	2.3
Indiana	2.3
Maine	2.7
Maryland	2.8
Massachusetts	2.4
Michigan	2.8
Minnesota	2.2
Missouri	3.5
Montana	2.4
Nebraska	2.7
New Mexico	2.1
Oregon	2.5
Pennsylvania	3.3
Tennessee	3.2
Texas	2.3
Vermont	3.1
Virginia	2.6
Washington	2.2
Wisconsin	2.7
DoDDS	1.2
<b>Percent Below the National Average</b>	
Arkansas	4.6
Delaware	0.8
District of Columbia	0.6
Louisiana	3.3
Mississippi	3.8
Nevada	3.1
New Jersey	3.1
New York	3.0
Rhode Island	2.8
South Carolina	2.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996  
Mathematics Assessment.

**Figure B2.7**

**Standard Errors for Percentage of Grade 8 Students Whose Teachers  
Had Professional Development in Use of Technology, for the Nation  
and the States: Public School Only, 1996**



<b>Grade 8</b>	
<b>Nation</b>	<b>2.9</b>
<b>Percent Above the National Average</b>	
Connecticut	2.1
Florida	3.2
Kentucky	1.7
Nebraska	1.7
North Carolina	1.8
North Dakota	1.2
Texas	2.3
Virginia	2.1
Wyoming	0.7
DDESS	1.1
DoDDS	0.5
<b>Percent Does Not Differ from the National Average</b>	
Alabama	3.6
Alaska	2.2
Arizona	4.4
Arkansas	4.2
California	3.3
Colorado	2.5
Delaware	1.0
District of Columbia	0.9
Georgia	2.5
Indiana	4.0
Iowa	3.8
Maine	2.9
Maryland	3.2
Massachusetts	3.0
Michigan	3.4
Minnesota	2.9
Mississippi	3.5
Missouri	3.6
Montana	2.5
New Mexico	2.2
New York	3.2
Oregon	3.2
South Carolina	3.7
Tennessee	3.4
Utah	1.5
Vermont	2.4
Washington	3.1
West Virginia	2.4
Wisconsin	3.7
<b>Percent Below the National Average</b>	
Hawaii	1.1
Louisiana	4.0
Rhode Island	0.9
Guam	1.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

**Table B2.8:**

**Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, Grades 4 and 8, 1996**



		Teacher Had Professional Development in Past Five Years			
		Yes		No	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>					
	All Students	2.3	1.2	2.3	1.5
<b>Grade 8:</b>					
	All Students	3.3	1.6	3.3	1.7
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	4.6	2.1	4.6	2.7
	Pre-Algebra	4.9	1.9	4.9	2.2
	Algebra	3.9	2.4	3.9	2.4

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessments.

Figure B2.8

**Standard Errors of Percentage of Grade 4 Students Whose Teachers  
Had Professional Development in Teaching Higher-Order Thinking  
Skills, for the Nation and States: Public Schools Only, 1996**

<b>Grade 4</b>	
<b>Nation</b>	<b>2.8</b>
<b>Percent Above the National Average</b>	
Maryland	2.8
North Carolina	3.6
Texas	2.5
<b>Percent Does Not Differ from the National Average</b>	
Arizona	3.4
Arkansas	4.3
California	3.6
Colorado	3.0
Connecticut	3.1
Delaware	1.2
Florida	3.2
Georgia	3.4
Hawaii	2.5
Indiana	4.2
Iowa	4.1
Kentucky	3.5
Louisiana	3.3
Massachusetts	3.7
Michigan	3.2
Minnesota	2.9
Mississippi	3.8
Montana	3.3
New Jersey	4.1
New York	3.3
Oregon	3.0
South Carolina	3.5
Tennessee	3.2
Utah	3.4
Virginia	3.5
Washington	3.6
DDESS	1.3
<b>Percent Below the National Average</b>	
Alabama	3.3
Alaska	3.3
District of Columbia	0.9
Maine	4.0
Missouri	3.4
Nebraska	3.7
Nevada	3.4
New Mexico	3.3
North Dakota	3.4
Pennsylvania	3.9
Rhode Island	2.6
Vermont	3.2
West Virginia	3.6
Wisconsin	3.5
Wyoming	3.7
Guam	1.5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996  
Mathematics Assessment.



Figure B2.9

**Standard Errors for Percentage of Grade 8 Students Whose Teachers Had Professional Development in Teaching Higher-Order Thinking Skills, for the Nation and States: Public Schools Only, 1996**

<b>Grade 8</b>	
<b>Nation</b>	<b>3.5</b>
<b>Percent Above the National Average</b>	
California	3.1
Maryland	4.5
North Carolina	3.3
Texas	3.4
DDESS	1.5
<b>Percent Does Not Differ from the National Average</b>	
Alabama	4.7
Arizona	3.4
Arkansas	4.0
Connecticut	3.1
Delaware	1.1
District of Columbia	1.3
Florida	3.3
Georgia	3.4
Hawaii	0.9
Indiana	4.2
Iowa	4.3
Kentucky	4.3
Louisiana	4.8
Maine	4.0
Massachusetts	4.1
Michigan	4.3
Minnesota	3.2
Mississippi	3.8
Missouri	4.0
Montana	3.5
North Dakota	2.6
South Carolina	4.0
Tennessee	4.0
Utah	1.5
Vermont	3.3
Virginia	3.3
Washington	3.9
West Virginia	3.0
Wisconsin	4.1
<b>Percent Below the National Average</b>	
Alaska	2.9
Colorado	2.9
Nebraska	2.9
New Mexico	1.8
New York	3.8
Oregon	3.7
Rhode Island	0.9
Wyoming	1.3
Guam	1.5
DoDDS	0.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B2.9

**Standard Errors for Percentage of Students and Average Scale Score by Number of College Courses in Mathematics or Mathematics Education Teachers Have Taken During the Last Two Years, Grades 4 and 8, 1996**



		Number of College Courses					
		Three or More Courses		One or Two Courses		No Courses	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>							
	All Students	1.0	3.2	1.9	2.8	2.2	1.1
<b>Grade 8:</b>							
	All Students	1.8	3.2	2.2	3.1	2.7	1.5
<b>Students Enrolled in:</b>							
	Eighth-Grade Mathematics	2.4	3.4	2.7	3.6	3.8	2.0
	Pre-Algebra	2.2	4.1	3.2	2.9	3.7	1.9
	Algebra	2.6	4.9	3.2	4.4	3.8	2.2

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B2.10

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Knowledge of NCTM Curriculum and Evaluation Standards, Grades 4 and 8, 1996**



		Level of Knowledge							
		Very Knowledgeable		Knowledgeable		Somewhat Knowledgeable		Little Knowledgeable	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>									
	All Students	1.1	4.5	1.9	1.9	2.1	1.5	2.3	1.5
<b>Grade 8:</b>									
	All Students	2.4	2.2	3.5	2.1	2.9	2.7	2.4	2.3
<b>Students Enrolled in:</b>									
	Eighth-Grade Mathematics	2.8	3.5	5.7	3.5	4.0	2.7	3.5	3.6
	Pre-Algebra	3.6	3.3	3.4	2.8	3.9	2.5	2.7	3.1
	Algebra	2.9	3.0	4.0	2.9	4.0	4.3	3.0	3.2

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

**Figure B3.1**

**Standard Errors for Percentage of Twelfth-Grade Students  
by Mathematics Graduation Requirement  
(Grades 9 through 12), 1996**



4 Years of Mathematics	3 Years of Mathematics	2 Years of Mathematics	One Year or Less of Mathematics
1.0	3.5	3.5	---

--- Standard error estimates cannot be accurately determined.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

**Table B3.1**

**Standard Errors for Percentage of Twelfth-Grade and Average  
Scale Score by Highest Algebra-Calculus Course Taken and  
Mathematics Graduation Requirements  
(Grades 9 through 12), 1996**



	Mathematics Graduation Requirement			
	3 or 4 Years		2 Years or Less	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Highest Algebra-Calculus Course Taken:				
Not Taken Pre-Algebra	---	6.2	0.7	4.2
Pre-Algebra	0.3	3.4	0.5	3.6
First-Year Algebra	1.4	2.0	1.5	2.4
Second-Year Algebra	2.1	1.6	2.2	1.4
Pre-Calculus/Third-Year Algebra	1.5	1.5	1.3	2.0
Calculus	1.5	2.8	1.2	4.0

--- Standard error estimates cannot be accurately determined.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B3.2

**Standard Errors for Percentage of Twelfth-Grade Students Who Have Taken Geometry by Mathematics Graduation Requirement (Grades 9 through 12), 1996**



Mathematics Graduation Requirement			
3 or 4 Years		2 Years or Less	
Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
1.8	1.6	2.2	1.4

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B3.3

**Standard Errors for Percentage of Twelfth-Grade Students and Average Scale Score by Whether Specific Advanced Mathematics Courses of One Semester in Length Taught in Their School**



	Offered		Not Offered	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Trigonometry</b>				
1996	3.0	1.1	3.0	3.1
1992	3.1	1.0	3.1	2.3
<b>Pre-Calculus, Third-Year Algebra, Elementary Functions, Analysis</b>				
1996	2.1	1.1	2.1	2.3
1992	2.2	1.0	2.2	3.0
<b>Calculus</b>				
1996	2.2	1.1	2.2	3.0
1992	2.3	1.1	2.3	2.0
<b>Probability and/or Statistics</b>				
1996	3.6	2.3	3.6	1.2
1992	3.1	1.8	3.1	1.0

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

**Figure B3.2****Standard Errors for Percentage of Students Whose Schools Offer Algebra for High School Credit for Placement**

Assessment Year	Percentage of Students
1996	3.6
1992	3.4
1990	4.2

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics.

**Figure B3.3**

**Standard Errors for Percentage of Students in Schools that Offer Algebra for Eighth-Grade Students, for the Nation and States: Public Schools Only, 1996**



<b>Grade 8</b>	
<b>Nation</b>	<b>2.8</b>
<b>Percent Above the National Average</b>	
Connecticut	1.8
Florida	2.7
Massachusetts	2.6
North Carolina	2.4
Washington	2.3
West Virginia	2.0
DDESS	0.2
<b>Percent Does Not Differ from the National Average</b>	
Alaska	2.1
Arizona	2.7
Arkansas	7.5
California	4.0
Colorado	3.4
Delaware	0.3
District of Columbia	0.5
Georgia	3.1
Hawaii	0.4
Indiana	3.9
Iowa	4.1
Maine	3.2
Maryland	2.8
Michigan	4.5
Minnesota	4.6
Mississippi	3.8
Missouri	4.0
Nebraska	2.8
New Mexico	3.7
New York	4.8
Oregon	4.2
Rhode Island	0.3
South Carolina	***
Tennessee	4.5
Texas	2.7
Utah	2.5
Vermont	1.9
Virginia	***
Wisconsin	3.7
Wyoming	1.1
Guam	***
DODDS	0.6
<b>Percent Below the National Average</b>	
Alabama	5.4
Kentucky	4.7
Louisiana	5.2
Montana	4.1
North Dakota	2.8

\*\*\* Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B3.4

**Standard Errors for Percentage of Eighth-Grade Students  
by Mathematics Course Enrollment and Availability of  
Algebra, 1996**



Mathematics Course Enrolled in:	Algebra Offered for High School Credit/Placement			
	Yes		No	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Eighth-Grade Mathematics	2.6	1.5	6.4	4.2
Pre-Algebra	2.2	2.1	4.4	4.6
Algebra	2.2	1.9	2.8	5.6
Other Mathematics	0.7	5.3	1.2	***

\*\*\* Sample size is insufficient to permit a reliable estimate.

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996  
Mathematics Assessment.



Figure B3.4

**Standard Errors for Percentage of Students Enrolled in Algebra,  
for the Nation and the States: Public Schools Only, 1996**

<b>Grade 8</b>	
<b>Nation</b>	<b>1.6</b>
<b>Percent Above the National Average</b>	
Delaware	1.1
District of Columbia	1.2
Maryland	2.1
Massachusetts	2.3
Minnesota	2.0
Rhode Island	0.9
Utah	1.9
DoDDS	1.2
<b>Percent Does Not Differ from the National Average</b>	
Alabama	2.1
Alaska	1.6
Arizona	1.7
Arkansas	1.7
California	2.2
Colorado	1.3
Connecticut	1.5
Florida	1.6
Georgia	1.9
Indiana	1.5
Iowa	1.7
Kentucky	1.8
Maine	1.5
Michigan	2.5
Missouri	1.8
Montana	1.4
Nebraska	1.6
New Mexico	1.2
North Carolina	1.4
North Dakota	1.3
Oregon	1.8
South Carolina	1.9
Texas	1.8
Vermont	1.3
Virginia	1.3
Washington	1.6
West Virginia	1.3
Wisconsin	2.1
Wyoming	0.8
Guam	1.6
DDESS	1.8
<b>Percent Below the National Average</b>	
Hawaii	0.9
Louisiana	1.4
Mississippi	1.3
New York	1.4
Tennessee	1.5

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

Table B3.5

**Standard Errors for Percentage of Students and Average Scale Score by Time on Mathematics Instruction, Grades 4 and 8**



		Time Spent Weekly on Mathematics						
			Two and One-Half Hours or Less		More than Two and One-Half Hours, But Less than Four Hours		Four Hours or More	
		Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4								
All Students	1996	1.1	2.4	2.3	1.7	2.6	1.0	
	1992	0.8	3.2	1.8	1.9	2.1	1.0	
Grade 8								
All Students	1996	2.8	2.6	3.1	1.6	3.1	2.7	
	1992	1.9	3.6	2.6	1.4	2.8	2.0	
Students Enrolled in:								
Eighth-Grade Mathematics	1996	4.6	3.6	4.4	2.4	3.3	3.6	
	1992	2.2	3.9	3.3	1.7	2.9	2.4	
Pre-Algebra	1996	3.1	3.7	3.9	2.2	4.3	2.8	
	1992	2.6	3.2	4.5	1.6	4.5	3.7	
Algebra	1996	2.5	3.9	5.0	2.9	5.9	3.2	
	1992	3.1	4.7	3.5	2.4	4.0	2.8	

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

**Figure B3.5**

**Standard Errors for Percentage of Grade 4 Students with  
Four or More Hours per Week of Mathematics Instructions,  
for the Nation and States: Public Schools Only, 1996**

THE NATION'S  
REPORT  
CARD



<b>Grade 4<sup>a</sup></b>	
<b>Nation</b>	<b>2.8</b>
<b>Percent Above the National Average</b>	
Alabama	2.9
Georgia	2.2
Louisiana	2.7
Maryland	3.1
Nevada	2.9
Texas	2.6
West Virginia	2.8
DDESS	0.8
<b>Percent Does Not Differ from the National Average</b>	
Alaska	3.0
Arizona	3.1
California	3.4
Colorado	3.2
Connecticut	2.9
District of Columbia	0.7
Florida	2.7
Hawaii	2.9
Kentucky	4.0
Maine	3.7
Massachusetts	3.6
Michigan	3.5
Minnesota	3.2
Mississippi	3.0
Montana	3.9
Nebraska	4.2
New Jersey	3.9
New Mexico	3.4
North Carolina	3.1
North Dakota	3.1
Pennsylvania	3.7
Rhode Island	3.0
South Carolina	2.6
Tennessee	2.8
Utah	3.4
Virginia	3.3
Wisconsin	3.9
Wyoming	2.9
DODDS	1.8
<b>Percent Below the National Average</b>	
Arkansas	4.6
Delaware	1.2
Indiana	3.5
Iowa	4.5
Missouri	3.7
New York	3.3
Oregon	3.7
Vermont	3.6
Washington	3.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessment.

**Figure B3.6**

**Standard Errors for Percentage of Grade 8 Students with  
Four or More Hours per Week of Mathematics Instructions,  
for the Nation and States: Public Schools Only, 1996**



<b>Grade 8:</b>	
<b>Nation</b>	<b>3.5</b>
<b>Percent Above the National Average</b>	
Alabama	3.8
Georgia	3.6
Mississippi	3.8
North Carolina	3.5
Tennessee	4.3
Guam	1.5
DDESS	1.8
<b>Percent Does Not Differ from the National Average</b>	
Alaska	2.1
Arizona	3.7
Arkansas	4.1
California	3.7
Colorado	2.6
District of Columbia	1.1
Florida	3.2
Kentucky	3.9
Louisiana	4.1
Maryland	4.0
Massachusetts	3.1
Michigan	4.2
Minnesota	3.6
Missouri	4.5
Montana	3.0
Nebraska	3.1
New Mexico	2.6
North Dakota	2.8
Oregon	3.9
Rhode Island	0.8
South Carolina	3.7
Texas	3.4
Virginia	3.6
Washington	3.0
West Virginia	3.1
Wisconsin	4.0
<b>Percent Below the National Average</b>	
Connecticut	3.2
Delaware	1.1
Hawaii	1.0
Indiana	3.9
Iowa	3.8
Maine	2.6
New York	2.2
Utah	1.5
Vermont	2.1
Wyoming	1.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Assessment.

123

Table B4.1

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Reports on the Availability of Resources, Grades 4 and 8**



		Availability of Resources							
			I Get All the Resources I Need		I Get Most of the Resources I Need		I Get Some or None of the Resources I Need		
		Assessment Year	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	
Grade 4		All Students	1996	1.8	2.3	2.0	1.4	2.3	1.4
			1992	1.5	2.4	2.6	1.1	3.1	1.8
			1990	2.1	2.6	2.8	1.6	2.9	1.7
Grade 8		All Students	1996	2.8	2.4	3.0	1.6	2.4	2.4
			1992	2.0	3.1	2.2	1.1	1.7	1.3
			1990	2.1	3.0	3.9	2.0	3.9	2.9
Students Enrolled in:		Eighth-Grade Mathematics	1996	3.9	2.8	4.2	2.2	3.2	3.8
			1992	2.8	2.8	3.2	1.6	2.7	2.0
			1990	2.6	2.5	4.0	1.8	3.9	2.9
		Pre-Algebra	1996	3.2	4.2	4.1	2.2	3.6	2.7
			1992	2.0	2.7	3.2	1.4	2.7	3.0
			1990	3.4	7.8	6.7	2.7	6.5	4.7
		Algebra	1996	4.1	2.8	4.6	2.7	2.6	3.7
			1992	2.3	3.4	3.1	2.1	2.8	3.8
			1990	3.4	6.8	5.7	4.1	5.8	4.5

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessments.

**Figure B4.2**

**Standard Errors for Percentage of Students and Average Scale Score by Whether Teachers Have Access to Mathematics Curriculum Specialists, Grades 4 and 8**



		Have Access		Do Not Have Access	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>					
	1996	3.4	2.0	3.4	1.3
	1992	3.1	1.2	3.1	1.3
<b>Grade 8</b>					
	1996	3.7	2.0	3.7	1.4

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Trial State Assessment.

**Table B4.3**

**Standard Errors for Percentage of Students and Average Scale Score by Teachers' Designated Preparation Time per Week, Grades 4 and 8, 1996**



		Hours of Designated Preparation Time							
		Less than One Hour		One to Two Hours		Three to Four Hours		Five or More Hours	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4</b>									
	All Students	1.4	3.6	2.0	2.2	2.3	1.3	2.5	1.9
<b>Grade 8</b>									
	All Students	1.2	6.4	1.8	5.3	3.1	2.1	3.3	1.6

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Mathematics Trial State Assessment.

**Table B4.4**

**Standard Errors for Percentage of Students and Average Scale Score of Students by Access to School-Owned Calculators, Grades 4 and 8**



		Have Access		Do Not Have Access	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 4:</b>					
	1996	1.9	1.0	1.9	2.7
	1992	3.1	1.3	3.1	1.5
<b>Grade 8:</b>					
	1996	3.4	1.3	3.4	2.5
<b>Students Enrolled in:</b>					
	Eighth-Grade Mathematics	4.4	1.7	4.4	4.3
	Pre-Algebra	4.5	1.9	4.5	3.6
	Algebra	4.7	2.1	4.7	3.1

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessment.

**Table B4.5**

**Standard Errors for Percentage of Students and Average Scale Score by Access to Calculators for Mathematics Schoolwork, Grade 12 Enrolled in Mathematics**



		Have Access		Do Not Have Access	
		Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Assessment Year</b>					
	1996	0.4	1.1	0.4	2.7
	1992	0.5	0.9	0.5	2.4

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996, 1992 Mathematics Assessment.

Table B4.6

**Standard Errors for Percentage of Students and  
Average Scale Score by Use of Scientific and  
Graphing Calculators, Grades 8 and 12, 1996**



	Scientific Calculators		Graphing Calculators	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
<b>Grade 8:</b>				
All Students	2.1	1.3	1.1	2.7
<b>Students Enrolled in:</b>				
Eighth-Grade Mathematics	2.2	1.5	0.8	3.6
Pre-Algebra	2.9	1.9	2.3	5.4
Algebra	3.0	1.5	2.3	3.5
<b>Grade 12:</b>				
Students Enrolled in Mathematics Course	1.3	1.0	2.0	1.1

NOTE: Information in this table is for both public and nonpublic students.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 and 1992 Mathematics Assessment.



## Acknowledgments

The authors want to thank all those who contributed to the production of this report. Among those who work for the American Institutes for Research, special thanks go to Sarah Beaver, Michelle Bullwinkle, Susan Cole, Marian Eaton, Jerald Hamlet, Mary Jenn, Julia Mitchell, and Anna Williams.

We would also like to thank people at ETS who provided us guidance, data analysis support, and the benefit of their knowledge and years of experience in producing such reports. Specifically we thank Loretta Casalaina, Carol Errickson, David Freund, Jeffrey Haberstroh, Frank Jenkins, Chancey Jones, Edward Kulick, Stephen Lazar, John Mazzeo, and Patricia O'Reilly.

The NAEP 1996 mathematics assessment was funded through NCES. NCES staff, in particular, Peggy Carr and Arnold Goldstein worked closely and collegially with the authors to produce this report. We are indebted to them for their efforts to coordinate the activities of the many people who contributed to this report.

We want to thank members of the NAEP Mathematics and Science Standing Committee who helped in the developing the focus of the report and provided advice on which content would be of particular interest and import to policymakers and educators. Specifically we would like to thank Audrey Jackson, Paul Koehler, Mary Lindquist, Christopher Olsen, and Zalman Usiskin. We would also like to thank Ray Fields from the staff of the National Assessment Governing Board for his participation with the Standing Committee in contributing to the development of this report.

Many thanks are due to the numerous reviewers, both internal and external to NCES. The comments and critical feedback of the following reviewers are reflected in this report: Peggy Carr, Janis Brown, Mary Frase, Arnold Goldstein, Steven Gorman, Marilyn McMillen, Michael Ross, Suzanne Triplett, and Kelly Weddel from the National Center for Education Statistics; Mary Crovo and Sharif Shakrani from the National Assessment Governing Board; Zalman Usiskin from the University of Chicago; and Peter Behuniak from the Connecticut State Department of Education.

ISBN 0-16-049716-7



9 780160 497162

United States  
Department of Education  
Washington, DC 20208-5653

Official Business  
Penalty for Private Use, \$300

Postage and Fees Paid  
U.S. Department of Education  
Permit No. G-17

**Standard Mail (B)**



BEST COPY AVAILABLE



**U.S. DEPARTMENT OF EDUCATION**  
*Office of Educational Research and Improvement (OERI)*  
*Educational Resources Information Center (ERIC)*



## **NOTICE**

### **REPRODUCTION BASIS**

☐

This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

☒

This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").